

الف) $y = x^2 - 5 \rightarrow x^2 = y + 5 \quad x^2 \geq 0 \rightarrow y + 5 \geq 0 \rightarrow y \geq -5$

$R = [-5, +\infty)$

ب) $y = x^3 + 1 \rightarrow x^3 = y - 1 \rightarrow x = \sqrt[3]{y-1} \Rightarrow (-\infty, +\infty)$
 $R = \mathbb{R}$

الف) $y = x^2 - 4x + 4 \rightarrow y = (x-2)^2 + 2 \rightarrow (x-2)^2 = y - 2 \rightarrow x - 2 = \pm \sqrt{y-2} + 2$
 $x^2 - 4x + 4 + 2 \rightarrow y - 2 \geq 0 \Rightarrow y \geq 2 \Rightarrow R = [2, +\infty)$

ب) $y = x^2 - 2x + 1 \rightarrow x^2 - 2x + (1-y) = 0 \quad \Delta = 2^2 - 4(1-y) = 4 + 4y$
 $\Delta \geq 0 \Rightarrow 4 + 4y \geq 0 \rightarrow y \geq -1 \Rightarrow y \geq -\frac{1}{1} \Rightarrow R = [-1, +\infty)$

الف) $y = \frac{x^2 + 3}{x^2 - 2} \rightarrow x^2 + 3 = yx^2 - 2y \rightarrow x^2 - yx^2 = -2y - 3 \rightarrow x^2(1-y) = -2y - 3$
 $x^2 = \frac{-2y-3}{1-y} \rightarrow x = \pm \sqrt{\frac{-2y-3}{1-y}}$
 $R = (-\infty, \frac{1}{2}] \cup (1, +\infty)$

ب) $y = \frac{x^2 + 1}{|x| - 2} \rightarrow y|x| - 2y = x^2 + 1 \rightarrow |x|(y-2) = 1 + y \rightarrow |x| = \frac{1+y}{y-2}$
 $|x| \geq 0 \Rightarrow \frac{1+y}{y-2} \geq 0 \Rightarrow R = (-\infty, -2) \cup [-\frac{1}{2}, +\infty)$

$y = \frac{1}{x^2 - 4x} \rightarrow yx^2 - 4xy = 1 \rightarrow yx^2 - 4xy - 1 = 0 \quad \Delta = b^2 - 4ac$
 $\Delta = 16y^2 + 4y \geq 0 \Rightarrow 4y(4y+1) \geq 0$
 $4y(4y+1) \geq 0 \Rightarrow 4y+1 \geq 0 \rightarrow 4y \geq -1 \rightarrow y \geq -\frac{1}{4} \quad \text{یا} \quad 4y+1 \leq 0 \rightarrow y \leq -\frac{1}{4}$
 $4y \geq 0 \rightarrow y \geq 0 \quad \text{یا} \quad 4y \leq 0 \rightarrow y \leq 0$
 $R = (-\infty, -\frac{1}{4}] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 2 \xrightarrow{\text{min}} \begin{cases} -\frac{b}{2a} = \frac{4}{2} = 2 \\ y = -V \end{cases} R = [-V, +\infty)$

ب) $y = -x^2 + 4x + 2 \xrightarrow{\text{max}} \begin{cases} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ y = V \end{cases} R = (-\infty, V]$

الف) $y = \sqrt{x^2 - 4x + 2}$ $\xrightarrow[\min]{a > 0}$ $\begin{cases} -\frac{b}{2a} = \frac{4}{2} = 2 \\ y = -1 \end{cases}$ $\sqrt{[-1, +\infty)} \rightarrow R = [0, +\infty)$

ب) $y = \sqrt{-x^2 + 4x + 10}$ $\xrightarrow[\max]{a < 0}$ $\begin{cases} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ y = 14 \end{cases}$ $\sqrt{(-\infty, 14]} \rightarrow R = [0, 14]$

الف) $y = x^3 + 3x^2 + 2x + 1$ $\rightarrow R_f = \mathbb{R}$
 بزرگترین توان سه فرد است

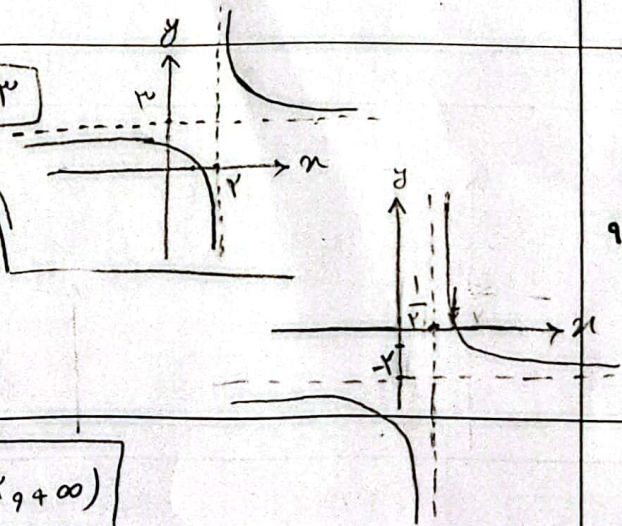
ب) $y = \sqrt{x^4 + 4x^2 + 4}$ $\rightarrow R_f = \mathbb{R}$
 بزرگترین توان فرد است

الف) $y = \frac{3x+1}{x-2}$ $R = \mathbb{R} - \{3\}$

ب) $y = \sqrt{\frac{4x+1}{x+3}}$ $\rightarrow \sqrt{\mathbb{R} - \{-1\}} \rightarrow R = [0, +\infty) - \{2\}$

الف) $y = \frac{3x+1}{x-2}$ $\boxed{x=2}$, $\boxed{y = \frac{a}{c} = 3}$

ب) $y = \frac{4x-2}{1-2x}$ $\boxed{x = \frac{1}{2}}$, $\boxed{y = \frac{a}{c} = \frac{4}{-2} = -2}$



الف) $y = \cos^2 x + \frac{1}{\cos^2 x}$ $\rightarrow \cos^2 x > 0$ $R = [2, +\infty)$

ب) $y = 3\sqrt{\frac{x^2+1}{x}}$ $\rightarrow x + \frac{1}{x} \rightarrow x > 0$ $\vee$ $x < 0$ $R = [2, +\infty) \cup (-\infty, -2]$
 ناکسری ندارد