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الف) $y = x^2 - 5 \rightarrow x^2 = y + 5 \quad x^2 \geq 0 \rightarrow y + 5 \geq 0 \rightarrow y \geq -5$

$R = [-5, +\infty)$ ✓

ب) $y = x^3 + 1 \rightarrow x^3 = y - 1 \rightarrow x = \sqrt[3]{y-1} \Rightarrow (-\infty, +\infty)$

✓ $R = \mathbb{R}$

الف) $y = x^2 - 4x + 4 \rightarrow y = (x-2)^2 + 2 \rightarrow (x-2)^2 = y - 2 \rightarrow x - 2 = \pm \sqrt{y-2} + 2$

$x^2 - 4x + 4 + 2 \quad \hookrightarrow y - 2 \geq 0 \Rightarrow y \geq 2 \quad R = [2, +\infty)$

ب) $y = x^2 - 2x + 1 \rightarrow x^2 - 2x + (1-y) = 0 \quad \Delta = 2^2 - 4(1-y) = 4 + 4y$

$\Delta \geq 0 \Rightarrow 4 + 4y \geq 0 \rightarrow y \geq -1 \rightarrow y \geq -\frac{1}{1} \quad R = [-1, +\infty)$

الف) $y = \frac{x^2 + 3}{x^2 - 2} \rightarrow x^2 + 3 = yx^2 - 2y \rightarrow x^2 - yx^2 = -2y - 3 \rightarrow x^2(1-y) = -2y - 3$

$R = (-\infty, \frac{1}{3}] \cup (1, +\infty)$ ✓ $x^2 = \frac{-2y-3}{1-y} \rightarrow x = \pm \sqrt{\frac{-2y-3}{1-y}}$

ب) $y = \frac{x^2 + 1}{|x| - 1} \rightarrow y|x| - y = x^2 + 1 \rightarrow |x|(y-1) = 1+y \rightarrow |x| = \frac{1+y}{y-1}$

$|x| \geq 0 \quad \hookrightarrow y + 1 \geq 0 \quad R = (-\infty, -1] \cup [-\frac{1}{2}, +\infty)$

$y = \frac{1}{x^2 - 4x} \rightarrow yx^2 - 4xy = 1 \rightarrow yx^2 - 4xy - 1 = 0 \quad \Delta = b^2 - 4ac$

$\hookrightarrow x(x-4) \rightarrow x \neq 0, 4 \quad \hookrightarrow (-4y)^2 - 4(y)(-1) = 16y^2 + 4y \geq 0$

$\hookrightarrow 4y(y+1) \geq 0 \quad \textcircled{1} \quad y+1 \geq 0 \rightarrow y \geq -1 \rightarrow y \geq -\frac{1}{1} \quad \textcircled{2} \quad y+1 \leq 0 \rightarrow y \leq -\frac{1}{1}$

$R = (-\infty, -\frac{1}{2}] \cup (0, +\infty)$ ✓ $4y \geq 0 \rightarrow y \geq 0 \quad 4y \leq 0 \rightarrow y \leq 0$

الف) $y = x^2 - 4x + 2 \xrightarrow{\text{min}} \begin{cases} -\frac{b}{2a} = \frac{4}{2} = 2 \\ y = -V \end{cases} R = [-2, +\infty)$

ب) $y = -x^2 + 4x + 2 \xrightarrow{\text{max}} \begin{cases} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ y = V \end{cases} R = (-\infty, 2]$

الف) $y = \sqrt{x^2 - 4x + 7}$ $\frac{a > 0}{\min} \rightarrow \begin{cases} -\frac{b}{2a} = \frac{4}{2} = 2 \\ y = -1 \end{cases} = \sqrt{[-1, +\infty)} \rightarrow R = [0, +\infty)$

ب) $y = \sqrt{-x^2 + 4x + 10}$ $\frac{a < 0}{\max} \rightarrow \begin{cases} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ y = 14 \end{cases}$ $\sqrt{(-\infty, 14]} \rightarrow R = [0, \sqrt{14}]$ ارفاق

الف) $y = x^3 + 3x^2 + 2x + 1$ $R_f = \mathbb{R}$
 بزرگترین توان سه فرد است

ب) $y = \sqrt{x^3 + 4x^2 + 4x + 1}$ $R_f = \mathbb{R}$
 بزرگترین توان فرد است

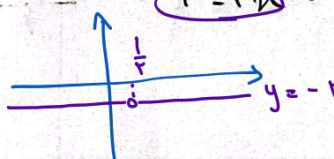
عبارت زیر رادیکال است
 پس قضا حاصل آن نامتناهی است
 $R_y = [0, +\infty)$

الف) $y = \frac{3x+1}{x-2}$ $R = \mathbb{R} - \{3\}$

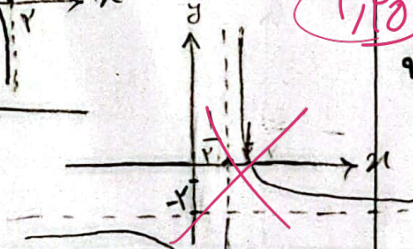
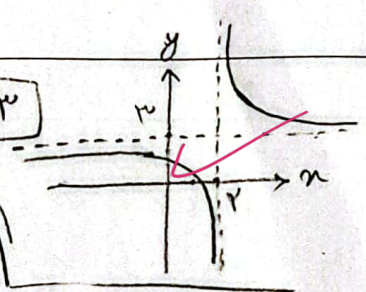
ب) $y = \sqrt{\frac{4x+1}{x+3}}$ $\rightarrow \sqrt{\mathbb{R} - \{-1\}} \rightarrow R = [0, +\infty) - \{2\}$

الف) $y = \frac{3x+1}{x-2}$ $x=2, y = \frac{a}{c} = 3$

ب) $y = \frac{4x-2}{1-2x}$ $x = \frac{1}{2}, y = \frac{a}{c} = \frac{4}{-1} = -4$



تجرباً $y = -2$ است



الف) $y = \cos^2 x + \frac{1}{\cos^2 x}$ $\rightarrow \cos^2 x > 0 \rightarrow R = [2, +\infty)$

ب) $y = 3\sqrt{\frac{x^2+1}{x}}$ $\rightarrow x + \frac{1}{x} \rightarrow x > 0, x < 0 \rightarrow R = [2, +\infty) \cup (-\infty, -2]$

$R_y = (-\infty, -\sqrt{2}] \cup [\sqrt{2}, +\infty)$