

الف) $y = x^r - a$
 $-x^r = -y - a \Rightarrow x^r = y + a \Rightarrow x = \pm \sqrt{y+a} \Rightarrow y \geq -a$
 $\Rightarrow RF = [-a, +\infty)$

ب) $y = x^r + 1$
 $-x^r = -y + 1 \Rightarrow x^r = y - 1 \Rightarrow x = \sqrt{y-1} \Rightarrow RF = [1, +\infty)$

الف) $y = x^r - \varepsilon x + r$
 $y = x^r - \varepsilon x + \varepsilon + r \Rightarrow y = (x-r)^r + r$
 $(x-r)^r = y-r \Rightarrow x-r = \pm \sqrt[r]{y-r} \Rightarrow x = r \pm \sqrt[r]{y-r} \Rightarrow y \geq r$
 $\Rightarrow RF = [r, +\infty)$

ب) $y = x^r - a x + 1 + \frac{r}{\varepsilon} - \frac{r}{\varepsilon}$
 $y = x^r - a x + 1 + \frac{r}{\varepsilon} - \frac{r}{\varepsilon} \Rightarrow y = (x - \frac{r}{a})^r - \frac{r}{\varepsilon}$
 $(x - \frac{r}{a})^r = y + \frac{r}{\varepsilon} \Rightarrow x - \frac{r}{a} = \pm \sqrt[r]{y + \frac{r}{\varepsilon}} \Rightarrow x = \frac{r}{a} \pm \sqrt[r]{y + \frac{r}{\varepsilon}} \Rightarrow y \geq -\frac{r}{\varepsilon}$
 $\Rightarrow RF = [-\frac{r}{\varepsilon}, +\infty)$

الف) $y = \frac{x^r + r}{x^r - r} \Rightarrow \frac{x^r + r}{x^r - r} = y$
 $x^r + r = y x^r - r y \Rightarrow x^r - y x^r = -r - r y \Rightarrow x^r (1-y) = -r - r y$
 $x^r = \frac{-r - r y}{1-y} \Rightarrow x = \sqrt[r]{\frac{r + r y}{y-1}}$
 $\Rightarrow RF = (-\infty, -1/a] \cup [1, +\infty)$

ب) $y = \frac{r|x|+1}{|x|-\varepsilon} \Rightarrow |x|y - \varepsilon y = r|x|+1 \Rightarrow |x|(y-\varepsilon) = \varepsilon y + 1$
 $|x|y - r|x| = \varepsilon y + 1 \Rightarrow |x| = \frac{\varepsilon y + 1}{y-r}$
 $\Rightarrow RF = (-\infty, -\frac{1}{\varepsilon}] \cup [r, +\infty)$

$y = \frac{1}{x^r - \varepsilon x} \Rightarrow \frac{1}{y} = x^r - \varepsilon x$
 $\frac{1}{y} = x^r - \varepsilon x + \varepsilon - \varepsilon \Rightarrow \frac{1}{y} = (x-r)^r - \varepsilon$
 $y = \frac{1}{(x-r)^r - \varepsilon} \Rightarrow y((x-r)^r - \varepsilon) = 1$
 $y(x-r)^r - \varepsilon y - 1 = 0$
 $-\varepsilon y - 1 = -y(x-r)^r$
 $\varepsilon y + 1 = y(x-r)^r$
 $\pm \sqrt[r]{\frac{\varepsilon y + 1}{y}} = x - r$
 $x = r \pm \sqrt[r]{\frac{\varepsilon y + 1}{y}}$
 $\Rightarrow RF = (0, +\infty) \cup (-\infty, -\frac{1}{\varepsilon}]$

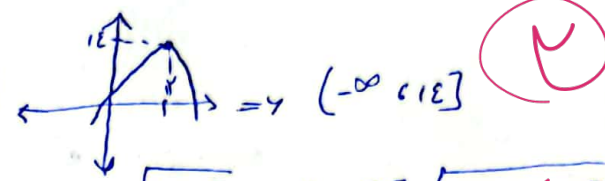
الف) $y = x^r - a x + r$
 $ext \quad a > 0 \Rightarrow \min$
 $\left. \begin{aligned} -\frac{b}{r a} &= \frac{r}{r} = 1 \\ a - 1a + r &= -1 \end{aligned} \right\} \Rightarrow RF = [-1, +\infty)$

ب) $y = -x^r + \varepsilon x + r$
 $ext \quad -1 < 0 \Rightarrow \max$
 $\left. \begin{aligned} -\frac{b}{r a} &= \frac{-\varepsilon}{-r} = r \\ -\varepsilon + 1a + r &= 1 \end{aligned} \right\} \Rightarrow RF = (-\infty, r]$

$\text{a)} \left\{ \begin{array}{l} x^2 - 4x + 4 \\ 140 = y \min \end{array} \right\} \text{ext} \left\{ \begin{array}{l} -\frac{D}{2A} = \frac{4}{2} = 2 \\ 4 - 16 + 4 = -8 \end{array} \right\} \Rightarrow y \in [-8, +\infty)$

$= y \in \mathbb{R} = [0, +\infty)$

$\text{b)} y = \sqrt{-x^2 + 4x + 10} \quad \text{ext} \left\{ \begin{array}{l} -\frac{D}{2A} = \frac{-4}{-2} = 2 \\ -4 + 16 + 10 = 22 \end{array} \right\} \Rightarrow y \in (-\infty, 11]$



$\sqrt{(-\infty, 11]} \Rightarrow y \in [0, 11] = \mathbb{R} \cap [0, 11]$

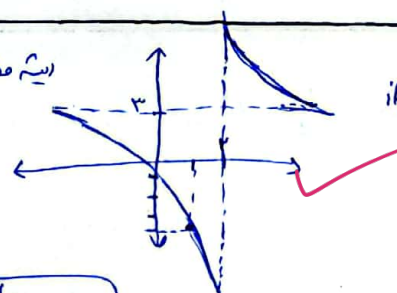
$\text{a)} y = x^2 + 2x + 1 \quad \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

$\text{b)} \sqrt{x^2 + 4x + 4} = y \in \mathbb{R} \cap \mathbb{R} = y \in \mathbb{R} = [0, +\infty)$

$\text{a)} y = \frac{x+1}{x-2} = y \in \mathbb{R} \cap \mathbb{R} = y \in \mathbb{R} \setminus \{2\}$

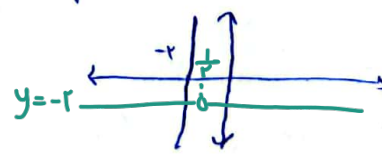
$\text{b)} y = \sqrt{\frac{x+1}{x-2}} = y \in \mathbb{R} \cap \mathbb{R} = y \in [0, +\infty) \setminus \{2\} = \mathbb{R}$

$\text{a)} \frac{x+1}{x-2} \quad \text{asymptotes: } x=2, y=1 \quad \text{if } x=1 \Rightarrow \frac{f}{-1} = -1$



$\text{b)} y = \frac{x-2}{1-2x} = y \in \mathbb{R} \cap \mathbb{R} = y \in \mathbb{R} \setminus \{1/2\}$

← برای معادله ها است چون بازای هر x
 جز 1/2 و همان 1/2 مرشود



$\text{a)} \cos^2 x + \frac{1}{\cos^2 x} \Rightarrow x + \frac{1}{x} \quad \text{if } x > 0 \Rightarrow y \in \mathbb{R} \cap [2, +\infty)$

$\text{b)} y = \sqrt{\frac{x^2+1}{x}} = y \in \mathbb{R} \cap \mathbb{R} \Rightarrow y^2 = x + \frac{1}{x} \quad y^2 \geq 2 \Rightarrow y \in \mathbb{R} \cap [2, +\infty) \cup (-\infty, -2]$