

1. د، و

الف)  $y = x^2 - \omega$

$y + \omega = x^2 \rightarrow \pm \sqrt{y + \omega} = x \Rightarrow D_f = [-\omega, +\infty)$

ب)  $y = x^3 + 1$

$y - 1 = x^3 \rightarrow \sqrt[3]{y - 1} = x \Rightarrow D_f = \mathbb{R}$

(2)

سوال (2)

الف)  $y = x^2 - kx + 4$

$y = (x^2 - kx + k^2/4) + 4 - k^2/4 \Rightarrow y = (x - k/2)^2 + 4 - k^2/4$   
 $(x - k/2)^2 = y - 4 + k^2/4$

$x - k/2 = \pm \sqrt{y - 4 + k^2/4} \Rightarrow x = \pm \sqrt{y - 4 + k^2/4} + k/2 \rightarrow D_f = [4 - k^2/4, +\infty)$

(2)

ب)  $y = x^2 - \omega x + 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \Delta = \omega^2 - 4 = 0$   
 $\Rightarrow x = \frac{\omega \pm \sqrt{\omega^2 - 4}}{2}$

$x = \frac{-b}{2a} = \frac{\omega}{2} = r, \omega$

$\hookrightarrow y = \frac{\omega^2}{4} - \frac{\omega^2}{2} + 1 = 1 - \frac{\omega^2}{4} = \frac{4 - \omega^2}{4}$

$D_f = [-1, +\infty)$

سوال (3)

الف)  $y = \frac{x^2 + 3}{x^2 - 1}$

$yx^2 - 1y = x^2 + 3$

$yx^2 - x^2 = 3 + y \rightarrow x^2(y - 1) = 3 + y \Rightarrow x^2 = \frac{3 + y}{y - 1}$

$x = \pm \sqrt{\frac{3 + y}{y - 1}}$

$3 + y = 0 \rightarrow y = -3$   
 $y - 1 = 0 \rightarrow y = 1$

$\frac{-1}{2} \quad 1$   
 $\frac{+}{-} \frac{+}{-} \frac{+}{+}$

$D_f = (-\infty, -1] \cup [1, +\infty)$

(2)

ب)  $y = \frac{r|x| + 1}{|x| - r}$

$r|x| + 1 = y|x| - ry$

$r|x| - y|x| = -ry - 1$

$|x|(r - y) = -ry - 1 \rightarrow |x| = \frac{-ry - 1}{r - y}$

$D_f = (r, +\infty) \cup (-\infty, \frac{1}{r}]$

$\frac{1}{2} \quad r$   
 $\frac{+}{-} \frac{+}{-} \frac{+}{+}$

$-ry - 1 = 0 \rightarrow -ry = 1 \rightarrow y = -\frac{1}{r}$   
 $r - y = 0 \rightarrow y = r$

سوال (2)

$x^2y - \varepsilon xy - 1 = 0 \rightarrow x = \frac{\varepsilon y \pm \sqrt{14y^2 + \varepsilon y}}{2y}$

$14y^2 + \varepsilon y \geq 0$   
 $y \neq 0$

$y = \frac{1}{x^2 - rx}$

$x^2 - rx \neq 0$   
 $x(x - r) \neq 0 \rightarrow x \neq 0, x \neq r$

$\frac{-1}{2} \quad \frac{1}{r}$   
 $\frac{+}{-} \frac{+}{-} \frac{+}{+}$

$D_f = (r, +\infty) \cup (-\infty, 0)$

$R_f = (-\infty, -\frac{1}{\varepsilon}] \cup (0, +\infty)$

$$الف) y = x^2 - 4x + 2$$

$a > 0 \rightarrow \min$

$$\begin{cases} x_{\min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{\min} = -\frac{V}{4} \\ y = (2)^2 - 4(2) + 2 \end{cases}$$

$$R_f = [-\infty, +\infty)$$

$$ب) y = -x^2 + 4x + 2$$

$a < 0 \rightarrow \max$

$$\begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{\max} = 4 \end{cases}$$

$$\frac{-4 + 16 + 2}{4}$$

$$R_f = (-\infty, 4]$$

سوال ۹

$$الف) y = \sqrt{x^2 - 4x + 2}$$

$a > 0 \rightarrow \min$

$$\begin{cases} x_{\min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{\min} = -1 \end{cases}$$

$$R_f = \sqrt{[-V, +\infty)} \rightarrow (0, +\infty)$$

مداومتی جزئیات  
در این عدد است!

$$ب) y = \sqrt{-x^2 + 4x + 10}$$

$a < 0 \rightarrow \max$

$$\begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{\max} = 6 \end{cases}$$

$$R_f = [0, \sqrt{14}]$$

$$R_f = \sqrt{[-\infty, -2]} \rightarrow 0$$

$$y_{\max} = -4 + 10 = 6$$

سوال ۷

$$الف) y = x^3 + 3x^2 + 2x + 1$$

چون این یک تابع درجه ۳ است  
پس بردار  $R$  است.

$$R_f = \mathbb{R}$$

$$ب) y = \sqrt{x^3 + 4x^2 + 4x + 1}$$

چون عبارت زیر رادیکال است و زیر رادیکال  
مربعوار، پس بردار  $R$  است.

$$R_f = [0, +\infty)$$

سوال ۸

$$الف) y = \frac{3x+1}{x-2}$$

$$\Rightarrow y = \frac{ax+b}{cx+d}$$

$$\Rightarrow R_f = \mathbb{R} - \left\{ \frac{a}{c} \right\} \Rightarrow R_f = \mathbb{R} - \left\{ \frac{3}{1} \right\}$$

$$ب) y = \sqrt{\frac{4x-2}{1-2x}}$$

چون این یک عبارت  
نسبی است و از  
استعداد گرفتن

$$R_f = [0, +\infty) - \left\{ \frac{1}{2} \right\}$$

$$\begin{aligned} 1-2x &\neq 0 \\ 1 &\neq 2x \\ \frac{1}{2} &\neq x \end{aligned}$$

$$R_y = \mathbb{R} - \left\{ \frac{3}{1} \right\}$$

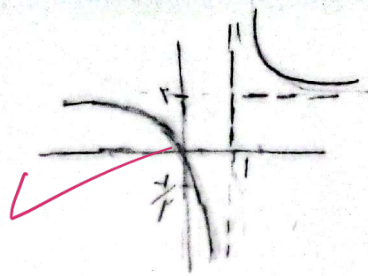
$\rightarrow$

$$[0, +\infty) - \left\{ \frac{1}{2} \right\}$$

1

الف)  $y = \frac{x+1}{x-2}$

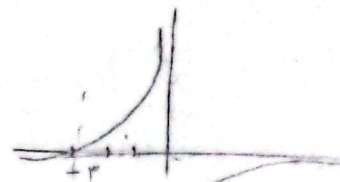
$\rightarrow$  جانب قائم  $\Rightarrow x=2$   
 جانب افقي  $\Rightarrow y = \frac{a}{c} = \frac{1}{1}$



$ad-bc$   
 $(-1) - (1) = -2 < 0$

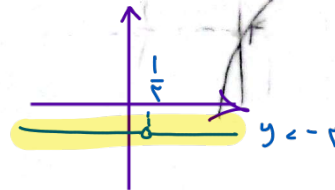
ب)  ~~$y = \sqrt{\frac{x+1}{x+3}}$~~

$\rightarrow$  جانب قائم  $\Rightarrow x=-3$   
 جانب افقي  $\Rightarrow y = \frac{a}{c} = \frac{1}{1} = 1$



$ad-bc$   
 $1 - 1 = 0$

$y = \frac{x-2}{1-x} = \frac{2(x-1)}{1-x} = -2$



سوال 10

1, 25

الف)  $y = \cos^2 x + \frac{1}{\cos^2 x}$

$R_f = [1, +\infty)$

ب)  $y = \sqrt[3]{\frac{x^2+1}{x}} \Rightarrow \sqrt[3]{x + \frac{1}{x}} \Rightarrow x + \frac{1}{x} \geq 1 \Rightarrow R_f = [1, +\infty)$

آیات

$\int x > 0 \rightarrow R_y = [\sqrt{x}, +\infty)$   
 $\int x < 0 \rightarrow R_y = (-\infty, -\sqrt{x}]$