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الف) $y = x^2 - 4$

$$\rightarrow x^2 = y + 4$$

$$x = \sqrt{y+4} \Rightarrow y+4 \geq 0 \Rightarrow y \geq -4 \Rightarrow R_f = [-4, +\infty)$$

ب) $y = x^3 + 1$

$$x^3 = y - 1 \rightarrow x = \sqrt[3]{y-1} \quad R_f = \mathbb{R}$$

$$y = x^2 - 4x + 4$$

$$y = x^2 - 4x + 4 + 2$$

$$y = (x-2)^2 + 2$$

$$\rightarrow y - 2 = (x-2)^2$$

$$\sqrt{y-2} = x-2$$

$$\sqrt{y-2} + 2 = x \Rightarrow y-2 \geq 0 \Rightarrow y \geq 2 \rightarrow R_f = [2, +\infty)$$

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ب) $y = x^2 - 4x + 1 \xrightarrow{+4, 20} y + 4, 20 = x^2 - 4x + 4, 20$

$$\frac{-4x}{2x} = -2, 10 \xrightarrow{20} 20$$

$$y + 4, 20 = (x-2, 10)^2$$

$$\sqrt{y+4, 20} = x-2, 10$$

$$\sqrt{y+4, 20} + 2, 10 = x \rightarrow y+4, 20 \geq 0$$

$$y \geq -4, 20 \Rightarrow R_f = [-4, 20, +\infty)$$

الف) $y = \frac{x^2+3}{x^2-2} \rightarrow \frac{x^2+3}{x^2-2} = y \rightarrow x^2 = \frac{-3y-3}{y-1} \Rightarrow x^2 = \frac{3y+3}{y-1}$

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$$x = \sqrt{\frac{3y+3}{y-1}} \rightarrow \frac{3y+3}{y-1} \geq 0 \quad \begin{array}{c} -1, 0 \quad 1 \\ + \quad | \quad - \quad | \quad + \\ 0 \quad 1 \quad 0 \end{array} \quad R_f = (-\infty, -1, 0] \cup (1, +\infty)$$

ب) $y = \frac{2|x|+1}{|x|-2} \rightarrow \frac{2|x|+1}{|x|-2} = y \rightarrow |x| = \frac{-2y-1}{y-2} \rightarrow |x| = \frac{2y+1}{y-2}$

$$\frac{2y+1}{y-2} \geq 0 \quad \begin{array}{c} -\frac{1}{2} \quad 2 \\ + \quad | \quad - \quad | \quad + \\ 0 \quad 2 \quad 0 \end{array} \quad R_f = (-\infty, -\frac{1}{2}] \cup (2, +\infty)$$

الف) $y = \frac{1}{x^2-4x} \rightarrow y(x^2-4x) = 1$

$$y(x^2-4x+4-4) = 1$$

$$y((x-2)^2-4) = 1$$

$$(x-2)^2-4 = \frac{1}{y} \rightarrow (x-2)^2 = \frac{1}{y} + 4 \rightarrow x-2 = \sqrt{\frac{1}{y} + 4}$$

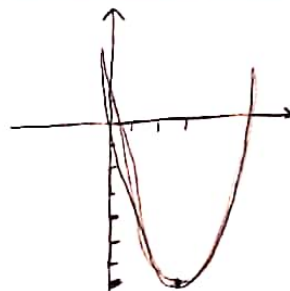
$$\rightarrow \frac{1}{y} + 4 \geq 0 \Rightarrow \frac{1}{y} \geq -4 \Rightarrow y \geq -\frac{1}{4}$$

$$R_f = [-\frac{1}{4}, +\infty)$$

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الف) $y = x^2 - 4x + 2 \quad \text{min} = \left| \frac{y}{x} \right| = r$

$$R_f = [-4, +\infty)$$



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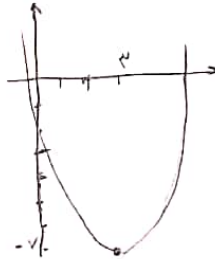
ج) $y = -x^2 + 4x + 2$ $\max \left| \frac{-\frac{4}{-2} = 2}{y} \right|$

$R_f = (-\infty, 4]$



د) $\sqrt{x^2 - 4x + 2}$ $\min \left| \frac{\frac{4}{2} = 2}{-2} \right|$

$R_f = \sqrt{[-2, \infty)} = [0, \infty)$



ج) $y = \sqrt{-x^2 + 4x + 1}$ $\max \left| \frac{-\frac{4}{-2} = 2}{1/2} \right|$

$R_f = \sqrt{[-\infty, 1/2]} = [0, \sqrt{1/2}]$



$-y = x^2 + 4x + 2x + 1$ $R_f = R_1$

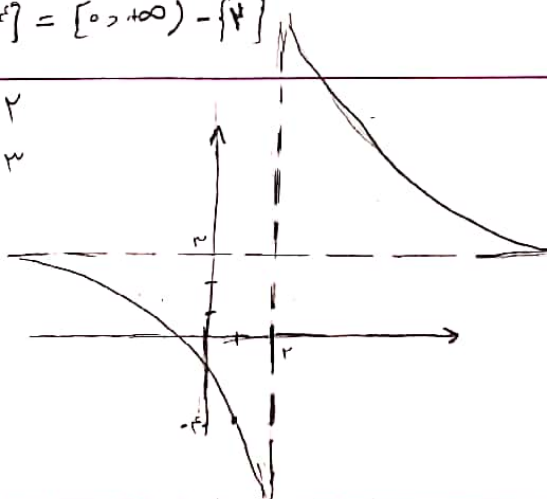
ج) $\sqrt{x^2 + 4x^2 + 4x + 1} \Rightarrow R_f = \sqrt{R_1} = [0, \infty)$

د) $y = \frac{4x+1}{2-x}$ $R_f = R_1 - \{2\} \Rightarrow R_f = R_1 - \{\frac{2}{4}\}$

ج) $y = \sqrt{\frac{4x+1}{x+2}}$ $R_f = \sqrt{R_1 - \{2\}} = [0, \infty) - \{2\}$

د) $y = \frac{4x+1}{x-2}$ $\begin{matrix} \text{میان } 2 = 2 \\ \text{میان } 2 = 2 \end{matrix}$

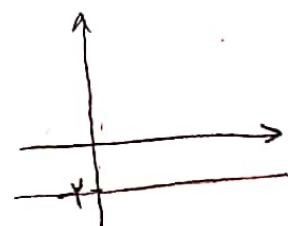
$x=1 \Rightarrow y = \frac{5}{-1} = -5$



ج) $y = \frac{4x-2}{-4x+1}$ $\begin{matrix} ad=8 \\ bc=4 \end{matrix} \Rightarrow \begin{matrix} ad=bc \\ \text{میان } 2 = 2 \end{matrix}$

$y = -2$

$y = \frac{4x-2}{-4x+1} = \frac{2(2x-1)}{-(4x-1)} = -2$



$$\text{الف) } y = \cos^r x + \frac{1}{\cos^r x} \quad R_f = [r, +\infty)$$

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$$\text{ب) } y = \sqrt[r]{\frac{x^r + 1}{x}} \rightarrow y = \sqrt[r]{x + \frac{1}{x}} \rightarrow y^r = x + \frac{1}{x} \rightarrow \begin{matrix} y^r \geq r \\ y^r \leq -r \end{matrix} \rightarrow \begin{matrix} y \geq \sqrt[r]{r} \\ y \leq \sqrt[r]{-r} \end{matrix}$$

$$R_f = [\sqrt[r]{r}, +\infty) \cup (-\infty, \sqrt[r]{-r}]$$