

-1

الف) $y = x^2 - 4$

$x^2 = y + 4$

$x = \sqrt{y+4} \Rightarrow y+4 \geq 0$
 $y \geq -4 \Rightarrow R_f = [-4, +\infty)$

1

ب) $y = x^2 + 1$

$x^2 = y - 1 \rightarrow x = \sqrt{y-1} \quad R_f = R$

$y = x^2 - 4x + 4$

$y = x^2 - 4x + 4 + 2$

$y = (x-2)^2 + 2$

$y - 2 = (x-2)^2$

$\sqrt{y-2} = x-2$

$\sqrt{y-2} + 2 = x \Rightarrow y-2 \geq 0$
 $y \geq 2 \rightarrow R_f = [2, +\infty)$

2

ب) $y = x^2 - 4x + 1$

$y + 4, 25 = x^2 - 4x + 4, 25$

$\frac{-4x}{2x} = -2, 5 \rightarrow x \geq 0$

$y + 4, 25 = (x-2, 5)^2$
 $\sqrt{y+4, 25} = x-2, 5$

$\sqrt{y+4, 25} + 2, 5 = x \rightarrow y+4, 25 \geq 0$

$y \geq -4, 25 \Rightarrow R_f = [-4, 25, +\infty)$

الف) $y = \frac{x^2+3}{x^2-2}$

$\frac{x^2+3}{x^2-2} = y \rightarrow x^2 = \frac{y-3}{y-1}$

$x^2 = \frac{y-3}{y-1} \Rightarrow x^2 = \frac{y-3}{y-1}$

$x = \sqrt{\frac{y-3}{y-1}}$

$\frac{y-3}{y-1} \geq 0$

$\frac{-1, 5}{0} \quad \frac{1}{1}$

$R_f = (-\infty, -1, 5] \cup (1, +\infty)$

3

ب) $y = \frac{2|x|+1}{|x|-2}$

$\frac{2|x|+1}{|x|-2} = y \rightarrow |x| = \frac{-2y-1}{y-2}$

$|x| = \frac{-2y-1}{y-2} \rightarrow |x| = \frac{-2y-1}{y-2}$

$\frac{-2y-1}{y-2} \geq 0$

$\frac{-1}{0} \quad \frac{2}{2}$

$R_f = (-\infty, -\frac{1}{2}] \cup (2, +\infty)$

4

الف) $y = \frac{1}{x^2-4x}$

$y(x^2-4x) = 1$

$y(x^2-4x+4-4) = 1$

$y((x-2)^2-4) = 1$

$y((x-2)^2-4) = 1$

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$\frac{1}{y} + 4 \geq 0 \Rightarrow \frac{1}{y} \geq -4 \Rightarrow y \geq -\frac{1}{4}$
 $R_f = [-\frac{1}{4}, +\infty)$

5

1

$x = \frac{4y \pm \sqrt{14y^2+5y}}{2y}$

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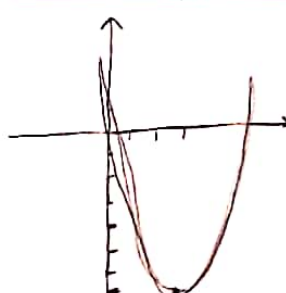
$x = \frac{4y \pm \sqrt{14y^2+5y}}{2y}$

$x = \frac{4y \pm \sqrt{14y^2+5y}}{2y}$

الف) $y = x^2 - 4x + 2$

$\min = \left| \frac{y}{-v} \right|$

$R_f = [-v, +\infty)$



2

ج) $y = -x^2 + 4x + 2$ $\max \left| \frac{-\frac{4}{-2} = 2}{y} \right|$

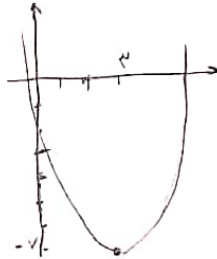
$R_f = (-\infty, 4]$ ✓



-2

ا) $\sqrt{x^2 - 4x + 2}$ $\min \left| \frac{\frac{4}{2} = 2}{-2} \right|$

$R_f = \sqrt{[-2, \infty)} = [0, \infty)$ ✓



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(2)

ج) $y = \sqrt{-x^2 + 4x + 1}$ $\max \left| \frac{-\frac{4}{-2} = 2}{1/2} \right|$

$R_f = \sqrt{[-\infty, 1/2]} = [0, \sqrt{1/2}]$ ✓



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$-y = x^2 + 4x + 2x + 1$ $R_f = R_1$ ✓

ج) $\sqrt{x^2 + 4x^2 + 4x + 1} \Rightarrow R_f = \sqrt{R_1} = [0, \infty)$ (2)

ا) $y = \frac{4x+1}{2-x}$ $R_f = R_1 - \{2\} \Rightarrow R_f = R_1 - \{\frac{2}{4}\}$ ✓

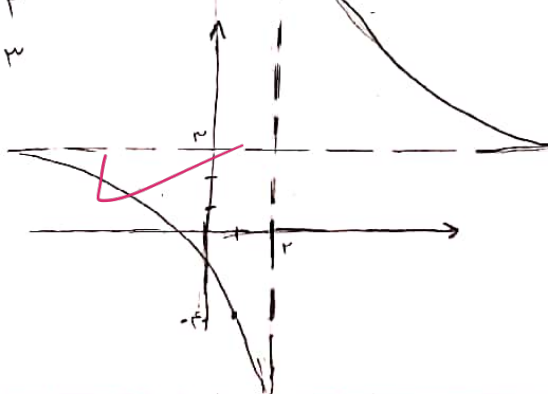
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ج) $y = \sqrt{\frac{4x+1}{x+2}}$ $R_f = \sqrt{R_1 - \{2\}} = [0, \infty) - \{2\}$ (2) ✓

ا) $y = \frac{4x+1}{x-2}$ $\begin{matrix} \text{مجاہد} = 2 \\ \text{مجاہد} = 2 \end{matrix}$

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$x=1 \Rightarrow y = \frac{5}{-1} = -5$

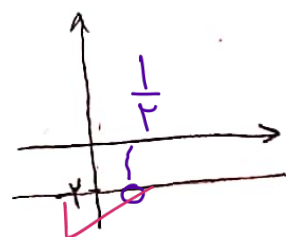


ج) $y = \frac{4x-2}{-4x+1}$ $\begin{matrix} ad=4 \\ bc=-4 \end{matrix} \Rightarrow \begin{matrix} ad=bc \\ \text{مجاہد} \end{matrix}$ $y = -2$

$y = \frac{4x-2}{-4x+1} = \frac{2(2x-1)}{-2(2x-1)} = -2$

$\Rightarrow 1-2x \neq 0 \Rightarrow x \neq \frac{1}{2}$

انفاق



(2)

$$\text{الف) } y = \cos^r x + \frac{1}{\cos^r x} \quad R_f = [r, +\infty)$$

٢ - ١٥

$$\text{ب) } y = \sqrt[r]{\frac{x^r + 1}{x}} \rightarrow y = \sqrt[r]{x + \frac{1}{x}} \rightarrow y^r = x + \frac{1}{x} \rightarrow \begin{matrix} y^r \geq r \\ y^r \leq -r \end{matrix} \rightarrow \begin{matrix} y \geq \sqrt[r]{r} \\ y \leq \sqrt[r]{-r} \end{matrix}$$

$$R_f = [\sqrt[r]{r}, +\infty) \cup (-\infty, \sqrt[r]{-r}]$$