①

الف) $y=x^2-5 \longrightarrow y+5=x^2 \longrightarrow x=\pm\sqrt{y+5}$

$y+5\geqslant 0 \rightarrow y\geqslant -5 \qquad R_f=[-5,+\infty)$

ب) $y=x^3+1 \longrightarrow y-1=x^3 \longrightarrow x=\sqrt[3]{y-1}$

$R_f=\mathbb{R}$

②

الف) $\frac{-b}{2a}=\frac{4}{2}=2 \qquad y=2^2-8+6=2 \qquad a>0 \qquad R_f=[2,+\infty)$

ب) $\frac{-b}{2a}=\frac{5}{2} \qquad y=(\frac{5}{2})^2-5(\frac{5}{2})+1=\frac{25}{4}-\frac{50}{4}+\frac{4}{4}=\frac{-21}{4} \qquad a>0 \longrightarrow R_f=[\frac{-21}{4},+\infty)$

③

الف) $yx^2-2y=x^2+3 \longrightarrow yx^2-x^2=2y+3 \longrightarrow x^2(y-1)=2y+3 \longrightarrow x^2=\frac{2y+3}{y-1} \longrightarrow x=\pm\sqrt{\frac{2y+3}{y-1}}$

$\frac{2y+3}{y-1}\geqslant 0 \longrightarrow \begin{array}{ccccc} & \frac{-3}{2} & & 1 & \\ + & \phi & - & \emptyset & + \end{array} \longrightarrow R_f=(-\infty,\frac{-3}{2}]\cup(1,+\infty)$

ب) $y|x|-4y=2|x|+1 \longrightarrow y|x|-2|x|=4y+1 \longrightarrow |x|(y-2)=4y+1 \longrightarrow |x|=\frac{4y+1}{y-2} \longrightarrow \qquad 0\leqslant\frac{4y+1}{y-2}$

$x=\frac{4y+1}{y-2} \longrightarrow \begin{array}{ccccc} & \frac{-1}{4} & & 2 & \\ + & \phi & - & \emptyset & + \end{array} \longrightarrow R_f=(-\infty,\frac{-1}{4}]\cup(2,+\infty)$

④

$y=\frac{1}{x^2-4x} \longrightarrow yx^2-4xy=1 \longrightarrow yx^2-4xy-1=0$

$\Delta\geqslant 0 \longrightarrow 16y^2-4(-1)(y)=16y^2+4y \longrightarrow x=y(16y+4)$ $\qquad \begin{array}{ccccc} & \frac{-1}{4} & & 0 & \\ + & \phi & - & \phi & + \end{array}$

$\longrightarrow R_f=(-\infty,\frac{-1}{4}]\cup[0,+\infty)$

⑤

الف) $x^2-6x+2 \qquad a>0 \longrightarrow Min \qquad Min\left|\begin{array}{l}\frac{-b}{2a}=\frac{6}{2}=3\\ 9-18+2=-7\end{array}\right.$

$R_f=[-7,+\infty)$

ب) $-x^2+4x+2 \longrightarrow a<0 \longrightarrow Max \qquad Max\left|\begin{array}{l}\frac{-b}{2a}=\frac{-4}{-2}=2\\ -4+8+2=6\end{array}\right.$

$R_f=(-\infty,6]$

الف) $y=\sqrt{x^2-6x+2}$ $\quad a>0 \to min \quad$ min $\left| \begin{array}{l} \frac{-b}{2a}=\frac{6}{2}=3 \\ 9-18+2=-7 \end{array}\right.$

$R_f=\sqrt{[-7,+\infty)} \to [0,+\infty)$

ب) $y=\sqrt{-x^2+4x+10}$ $\quad a<0 \to max \quad$ max $\left| \begin{array}{l} \frac{-b}{2a}=\frac{-4}{-2}=2 \\ -4+8+10=14 \end{array}\right.$

$R_f=\sqrt{(-\infty,14]} \to [0,\sqrt{14}]$

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الف) بزرگترین توان ← فرد $\quad R_f=R$

ب) بزرگترین توان ← فرد $\quad R_f=\sqrt{R}=[0,+\infty)$

8

الف) $y=\frac{3x+1}{x-2} \to R_f=R-\{3\}$

ب) $y=\sqrt{\frac{4x+1}{x+3}} \to R_f=\sqrt{R-\{4\}} \to [0,+\infty)-\{2\}$

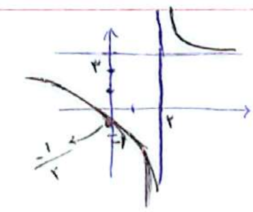
9

الف) $y=\frac{3x+1}{x-2}$

ریشه مخرج: مجانب قائم ← 2

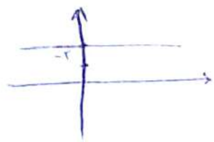
مجانب افقی: $\frac{a}{c}=3$

$x=0 \to \frac{-1}{2}$



ب) $y=\frac{4x-2}{1-2x}$

↳ $\frac{4x-2}{-(2x-1)}=-2$



10

الف) $y=\cos^2x+\frac{1}{\cos^2x} \to \cos^2>0 \quad R_f=[2,+\infty)$

ب) $y=\sqrt[3]{\frac{x^2+1}{x}} \to \sqrt[3]{x+\frac{1}{x}} \quad x>0$ یا $x<0$

↳ $R_f=\sqrt[3]{(-\infty,-2]\cup[2,+\infty)} \to R_f=(-\infty,\sqrt[3]{-2}]\cup[\sqrt[3]{2},+\infty)$