

الف) $f(x) = \sqrt{x|x|}$, $g(x) = x$ $D_f = \{x|x| \geq 0 \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty)$
 $D_g = \mathbb{R}$ چون دامنه ها یکسان نیست دو تابع با هم برابر نیستند

ب) $f(x) = \frac{(x+1)(x+1)+x+1}{x^2-x+1}$, $g(x) = 1$ $D_f = D_g = \mathbb{R}$ $f(x) = 1$ $g(x) = 1$ پس دو تابع با هم برابرند
 ج) $f(x) = \frac{x \sin x - \sin x}{x \sin x - x}$, $g(x) = \sin x$ $f(x) = \sin x$ $g(x) = \sin x$ دو تابع با هم برابرند

د) $f(x) = \frac{x}{|x|}$, $g(x) = \frac{|x|}{x}$ $D_f = D_g = \mathbb{R} - \{0\}$ با یکدیگر حاصل هر دو برابر می شود پس دو تابع برابرند

ه) $f(x) = \left[\frac{x^2}{x^2+1} \right]$, $g(x) = [x - [x]]$ $D_f = D_g = \mathbb{R}$ $f(x) = 0$, $g(x) = 0$ حاصل هر دو تابع همیشه با هم برابر است پس هر دو برابرند.

و) $f(x) = \frac{1}{[x]}$, $g(x) = \frac{1}{x}$ $D_f = \mathbb{R} - [0, \frac{1}{2})$, $D_g = \mathbb{R} - \{0\}$ دامنه ها مساوی نیست در نتیجه با هم برابر نیستند.

ز) $f(x) = \frac{1}{\sqrt{x-|x|}}$, $g(x) = \frac{1}{\sqrt{|x|-x}}$ $D_f = \emptyset$ $D_g = (-\infty, 0)$ دامنه ها مساوی نیست پس با هم برابر نیستند.

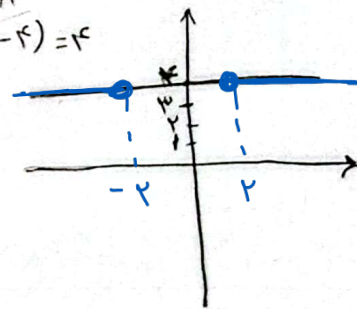
ح) $f(x) = \begin{cases} \frac{x^2-x}{x-1} & x \neq 1 \\ 2 & x = 1 \end{cases}$, $g(x) = \frac{x(x-1)}{x-1}$ $\rightarrow \frac{x(x-1)}{x-1} = \frac{x(x+1)(x-1)}{x-1} = g(x)$ هر دو تابع برابرند.

۱) $f^2(x) - g^2(x) = (f(x) - g(x))(f(x) + g(x)) = (x^2 - x + 1)(x^2 + x + 1) = x^4 + x^2 + 1$

۲) $\sqrt{f^2(x)} = \sqrt{x^4} = x^2$, $\sqrt{g^2(x)} = \sqrt{x^2} = |x|$, $x f(x) g(x) = 1 \Rightarrow (\pm 1)(\pm 1)x^2 = \frac{1}{x} \Rightarrow x^3 = \pm \frac{1}{x}$
 $x = \pm \sqrt[3]{\frac{1}{x}}$
 ۳) حالت اول: $f(x) = x^2 = \left(\sqrt[3]{\frac{1}{x}}\right)^2$, $g(x) = \frac{1}{x} \Rightarrow f(x) = \sqrt[3]{\frac{1}{x}}$ $g(x) = \sqrt[3]{\frac{1}{x}}$

۴) $f(x) \times g(x) = (x - \sqrt{x^2 - 4})(x + \sqrt{x^2 - 4}) \Rightarrow x^2 - (x^2 - 4) = 4$

$D_{f \times g} = x^2 - 2 \rightarrow x^2 \geq 2$
 $x \geq \sqrt{2} \leq x \leq -\sqrt{2}$



$g(x) = f(x)$ $\frac{x-b}{x^2-3x-6} = \frac{ax^2+c}{x^2-mx+n} \Rightarrow x^3 - mx^2 + nx - bx + bm - bn = ax^3 + (-3a+c)x^2 + (-6a-3c)x - 6c$

$\Rightarrow x^3 - (m+b)x^2 + (n+bm)x - bn = ax^3 + (-3a+c)x^2 + (-6a-3c)x - 6c$ ① $m = \frac{3}{2}$, $n = -\frac{9}{2}$, $am - bn = 10$
 ② $m = -\frac{3}{2}$, $n = -10$, $am - bn = 10$
 ③ $m = -\frac{3}{2}$, $n = \frac{9}{2}$, $am - bn = 10$
 $\Rightarrow a = \frac{1}{x^2 - (m+b)} = -\frac{1}{x^2 - (-\frac{3}{2} + \frac{3}{2})} = -\frac{1}{x^2} \Rightarrow m+b = -\frac{3}{2} \Rightarrow n+bm = -10 \Rightarrow \frac{9}{2} - 10 = -10 \Rightarrow \frac{9}{2} = 0$
 $m = -\frac{3}{2}$, $b = -\frac{3}{2}$, $am = \frac{1}{x^2 - (-\frac{3}{2} - \frac{3}{2})} = \frac{1}{x^2 - 3}$, $bn = -10$, $am - bn = 10$

$$f(x) = \frac{bx+r}{\lambda x+b}, g(x) = c, D_f = \mathbb{R} - \{a\} \quad bx+r = \lambda cx + bc \Rightarrow x(b-\lambda c) + r = bc$$

$$b-\lambda c = 0 \Rightarrow b = \lambda c \Rightarrow bc = r = \lambda c^2$$

$$\lambda x + b = 0 \Rightarrow x = \frac{b}{-\lambda} = a = -\frac{\lambda c}{\lambda} = -c \quad \Rightarrow c^2 = \frac{1}{\lambda} \Rightarrow c = \pm \frac{1}{\sqrt{\lambda}} / b = \pm \sqrt{\lambda}$$

$$ab = (-c)(\lambda c) = -\lambda c^2 \quad \text{① } c = \frac{1}{\sqrt{\lambda}} \Rightarrow \frac{ab}{c} = -\lambda \times \frac{1}{\sqrt{\lambda}} = -\sqrt{\lambda}$$

$$\hookrightarrow \frac{\lambda c^2}{c} = \lambda c, (c \neq 0) \quad \text{② } c = -\frac{1}{\sqrt{\lambda}} \Rightarrow \frac{ab}{c} = \sqrt{\lambda}$$

$$\Rightarrow \boxed{\frac{ab}{c} = \pm \sqrt{\lambda}}$$

$$f = \{(-1, 2), (2, 3), (3, -2), (0, -\frac{1}{2})\} \quad g = \{(2, 1), (3, 2), (-1, -1), (0, 0)\}$$

$$f+g = \{(1, 3), (5, 5), (-1, -2)\}$$

$$\frac{fg}{f+g} = \left\{ \left(1, \frac{1}{3}\right), \left(5, \frac{1}{5}\right), \left(-1, \frac{1}{-2}\right) \right\}$$

$$\frac{fg}{f+g} = \{(-1, \frac{1}{-2}), (1, \frac{1}{3}), (5, \frac{1}{5})\}$$

$$g = \{(1, -1), (-3, 1), (a-3b, d)\} = f = \{(1, a), (-3, \frac{a-3b}{-3-2b}), (c, d), (1, d)\}$$

$$a = -1 \quad \Rightarrow \frac{a-3b}{-3-2b} = \frac{1-3b}{-3-2b} \Rightarrow b = -2$$

$$a-3b = c \Rightarrow -1+6 = c \Rightarrow c = 5 \quad d = -1 \quad d+c = -1+5 = 4$$

$$-x^2 + x - m \geq 0 \Rightarrow x = \frac{-b}{2a} = \frac{-1}{-2} = \frac{1}{2} \Rightarrow \sqrt{-\frac{1}{4} + \frac{1}{4} - m} = \sqrt{\frac{1}{4} - m}$$

$$\frac{1}{4} - m = 0 \Rightarrow m = \frac{1}{4} \quad f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{4} + \frac{1}{4} - \frac{1}{4}} = \sqrt{0} = 0$$

$$(a, b) = \left(\frac{1}{2}, 0\right) \Rightarrow a+b = \frac{1}{2} + 0 = \frac{1}{2}$$

$$f(x) = \frac{px+a}{x^2+qx+b} \Leftrightarrow g(x) = \frac{p}{x-c} \quad a+b+c = ?$$

$$\hookrightarrow \Delta = 0 \Rightarrow b = 1 \quad x-c = x+3 \Rightarrow c = -3$$

$$px^2 - px + a - ac = px^2 + qx + b \Rightarrow (-p+c)x - ac = qx + b$$

$$\Rightarrow -p+c = q \quad / \quad -ac = b \quad \Rightarrow b = -3c - c^2$$

$$a = 4+3c \quad \Rightarrow a+b+c \Rightarrow (4+3c) + (-3c-c^2) + c \Rightarrow 4-c^2 \quad a+b+c = 12$$

$$\frac{px+a}{(x+3)^2} = \frac{p}{x+3} \rightarrow px+4 < px+a \rightarrow a=4$$