نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۸ کلاس دهم رشته B .....

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الف) $f(x)=\sqrt{x|x|}$ $x|x|\geq 0$ $\begin{cases} x\geq 0 \\ x<0 \end{cases}$ $\Rightarrow D_f=\mathbb{R}^+\cup\{0\}$ $D_g=\mathbb{R}$ $\Rightarrow$ برابر نیست

ب) $f(x)=\dfrac{x^2+4x+4+x+3}{x^2+5x+7}=\dfrac{x^2+5x+7}{x^2+5x+7}=1$ $g(x)=1$ $\Rightarrow$ برابر است

$x^2+5x+7\neq 0 \rightarrow \Delta=25-28=-3<0 \Rightarrow \times$ همیشه

ج) $2\sin x-1\neq 0$ $\sin x\neq \frac{1}{2}$ $\checkmark$ $-1\leq \sin x\leq 1 \Rightarrow D_f=\mathbb{R}$, $D_g=\mathbb{R}$ $\Rightarrow$ برابر است

د) $|x|\neq 0 \rightarrow x\neq 0 \Rightarrow D_f=\mathbb{R}-\{0\}$

$x\neq 0 \rightarrow D_g=\mathbb{R}-\{0\}$ $\Rightarrow$ برابر است

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الف) $f(x)\Rightarrow x^2+1\neq 0\checkmark$ $\dfrac{x^2}{x^2+1}$ $\begin{cases} x=0 \rightarrow [\frac{0}{1}]=0 \\ x\neq 0 \rightarrow \frac{x^2}{x^2+1}<1 \rightarrow [\frac{x^2}{x^2+1}]=0 \end{cases}$ $g(x)\Rightarrow x-[x]\rightarrow$ عبارت صفر یا مثبت

$0\leq x-[x]<1$

$[x-[x]]=0$ $\Rightarrow$ برابر است

ب) $f(x)\Rightarrow [2x]\neq 0$ $\Rightarrow D_f=\mathbb{R}-[0,\frac{1}{2})$, $g(x)\Rightarrow 2[x]\neq 0$ $[x]\neq 0$ $\Rightarrow D_g=\mathbb{R}-[0,1)$

$D_f\neq D_g$ $\Rightarrow$ برابر نیست

ج) $f(x)\rightarrow x-|x|>0$ $\begin{cases} x\geq 0 \rightarrow 0>0 \times \\ x<0 \rightarrow 2x>0 \Rightarrow x>0 \times \end{cases}$ $D_f=\emptyset$ $g(x)\Rightarrow |x|-x>0$ $\begin{cases} x=0 \Rightarrow 0>0 \times \\ x<0 \rightarrow -2x>0 \Rightarrow x<0\checkmark \end{cases}$ $\Rightarrow D_g=\mathbb{R}^-$

$D_f\neq D_g \Rightarrow$ برابر نیست

د) $g(1)=2$ $f(1)=2$ $g(x\neq 1)=x(x+1)=\dfrac{x(x-1)(x+1)}{(x-1)}$ $\Rightarrow$ برابر است

$g(x\neq 1)=x^2+x=x(x+1)$

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$f(x)+g(x)=x^2+x+1$

$f(x)-g(x)=x^2-x+1$

$f^2(x)-g^2(x)=(x^2+1)^2-x^2=$

$x^4+1+2x^2-x^2=x^4+1+x^2$

$2g(x)=2x \Rightarrow \boxed{g(x)=x}$

$f(x)+x=x^2+x+1$

$\boxed{f(x)=x^2+1}$

$\boxed{f^2(x)-g^2(x)=x^4+x^2+1}$

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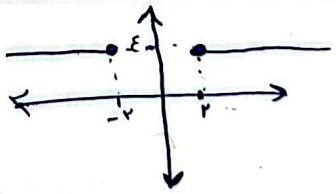
$f(x)=x-\sqrt{3-x^2}$

$g(x)=x+\sqrt{3-x^2}$



$f\times g=(x-\sqrt{3-x^2})(x+\sqrt{3-x^2})=x^2-(3-x^2)=2x^2-3$ $\Rightarrow f\times g=3$

$D_f=D_g=(-\infty,-\sqrt{3}]\cup[\sqrt{3},+\infty)$

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$f(x)=\dfrac{ax+2}{x^2-mx+n}$ $g(x)=\dfrac{x-b}{2x^2-3x-5}$ $f(x)=g(x)$

$g(x)\rightarrow 2x^2-3x-5\neq 0 \rightarrow (2x-5)(x+1)\neq 0 \rightarrow x\neq \frac{5}{2},-1 \Rightarrow D_g=\mathbb{R}-\{-1,\frac{5}{2}\}$

$f(x)\rightarrow (x+1)(x-\frac{5}{2})=x^2-\frac{3}{2}x-\frac{5}{2} \rightarrow m=\frac{3}{2}$ $n=-\frac{5}{2}$

$\dfrac{ax+2}{x^2-\frac{3}{2}x-\frac{5}{2}}=\dfrac{x-b}{2x^2-3x-5}$ $2ax+4=x-b$ $\xrightarrow{x=0}$ $4=-b \Rightarrow b=-4$

$x=1$ $2a+4=1+4=5 \Rightarrow a=\frac{1}{2}$

$am-bn=\frac{1}{2}\times\frac{3}{2}-(-4\times\frac{5}{4})=\boxed{\frac{37}{4}}$

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$f(x) = \frac{bx+2}{8x+b}$ $\quad g(x)=c \quad D_g = \mathbb{R}-\{a\}$

$D_f = \mathbb{R} - \{\frac{-b}{8}\} \Rightarrow D_f = D_g \Rightarrow a = \frac{-b}{8} \Rightarrow b = -8a = \{-4, 4\}$

$\frac{-8ax+2}{8x-8a} \xrightarrow{x=0} \frac{2}{-8a} = c \Rightarrow c = \frac{-1}{4a}$

① $\ldots = \boxed{4}$ ② $\ldots = \boxed{-4}$ $\rightarrow \boxed{\frac{ab}{c} = \pm 4}$

$x=1 \quad \frac{-8a+2}{8-8a} = \frac{-1}{4a} = c \quad 8a - 8 = -32a^2 + 8a \Rightarrow a^2 = \frac{1}{4} \begin{cases} a = \frac{1}{2} \\ a = -\frac{1}{2} \end{cases}$

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$2f-1 = \{(-1,3),(2,7),(3,-5),(0,0)\} \rightarrow 2f = \{(-1,4),(2,8),(3,-4),(0,1)\}$

$f = \{(-1,2),(2,4),(3,-2),(0,\frac{1}{2})\}$

$g = \{(-1,-4),(2,4),(3,6),(5,0)\}$

$\frac{2g}{f+g}$: $x=-1 \rightarrow \frac{-8}{-2} = 4$; $x=2 \rightarrow \frac{8}{8} = 1$; $x=3 \rightarrow \frac{12}{4} = 3$

$\boxed{\text{برد‌ها} = 4, 1, 3}$

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$f = \{(2,a),(-3,3a-2b),(c,5),(2,d)\}$ چون تابع $a=d$

$g = \{(2,-1),(-3,1),(a-3b,5)\}$

$a=-1 \quad -3-2b=1 \Rightarrow b=-2 \quad -1+6=c=5$

$d+c = a+c = -1+5 = \boxed{4}$

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$-(x^2-x+m) \geq 0 \Rightarrow$ تک‌جواب $\Rightarrow \Delta = 0 \Rightarrow$ ریشه مضاعف $\Rightarrow \sqrt{-(x-a)^2}$

$D_f = D_g \Rightarrow = \{a\}$

$-x^2+2ax-a^2 = -x^2+x-m \Rightarrow 2a=1 \Rightarrow \boxed{a=\frac{1}{2}} \Rightarrow \boxed{a+b=\frac{1}{2}}$

$-a^2 = -m \quad m = \frac{1}{4} \quad \boxed{b=0}$

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$x=0 \longrightarrow \frac{2}{-c} = \frac{a}{b} \Rightarrow 2b = -ac$

$\frac{2}{x-c} = \frac{2(x+\frac{a}{2})}{(x-c)(x+\frac{a}{2})} \Rightarrow \frac{a}{2} - c = 6 \Rightarrow a - 2c = 12$

$D_f = D_g \Rightarrow \mathbb{R} - \{c, -\frac{a}{2}\} = \mathbb{R} - \{c\} \Rightarrow c = -\frac{a}{2} \Rightarrow a = -2c$

$-2c-2c=12 \Rightarrow c=-3, \quad a=6, \quad b=9 \quad \boxed{a+b+c=12}$