سوال ۱

آرا حاتمی - دهم تجربی - 8

الف) $f(x) = \sqrt{x|x|}$ ①، $g(x) = x$ ②

② $x \geq 0 \Rightarrow [0, +\infty)$ دامنه‌ها برابر X

① $x|x| \geq 0 \Rightarrow [0, +\infty)$ یا $R^+ \cup \{0\}$

X

ب) $f(x) = \dfrac{(x+3)(x+1)+x+4}{x^2+5x+7}$ $g(x) = 1$ $g(x) = R$

$x^2+5x+7 > 0$

$x^2+5x > -7$

$x(x+5) > -7$

$-3 \quad -1$

$+ \mid - \mid +$

$g = [-1, +\infty) \cup [-3, -\infty)$

برابر ✓

ج) $f(x) = \dfrac{4\sin^2 x - 6\sin x}{2\sin x - 3}$ $g(x) = 2\sin x$ $g(x) = 2\sin x \geq 0$

$\sin x \geq \frac{1}{2}$

$f(x) = \dfrac{4\sin^2 x - 6\sin x}{2\sin x - 3} \geq 0 \to 4\sin^2 x - 6\sin x \geq 0$

$2\sin x (2\sin x - 3) \geq 0$

$2\sin x \geq 0 \to \sin x \geq \frac{1}{2}$

$2\sin x - 3 \geq 0$

$\sin x \geq \frac{3}{2}$

X برابر

د) $f(x) = \dfrac{x}{|x|}$ $g(x) = \dfrac{|x|}{x}$

$\dfrac{x}{|x|} \geq 0 \to x \geq 0$ $|x| > 0 \Rightarrow D_f = R - \{0\}$ $|x| \neq 0$

$\dfrac{|x|}{x} \geq 0 \to |x| \geq 0$ $x > 0 \Rightarrow D_g = R - \{0\}$ $x \neq 0$

برابر ✓

سوال ۲

۱) $f(x) = \left[\dfrac{x^2}{x^2+1}\right]$ $g(x) = [x - [x]]$ ✓ برابر $D_{f,g} = R$

$D_f = R$ $D_g = R$

۲) $f(x) = \dfrac{1}{[2x]}$ $g(x) = \dfrac{1}{2[x]}$ $[2x] > 0$

$[2x] = 0$

$D_f = [0, 1] - [\frac{1}{2}]$

$D_g = [0, 1)$ X برابر

$2[x] = 0$

$[x] = 0$

۳) $f(x) = \dfrac{1}{\sqrt{x - |x|}}$ $g(x) = \dfrac{1}{\sqrt{|x| - x}}$

$x - |x| > 0$ $|x| - x > 0$ X برابر

نادرست $x > |x| \Rightarrow D_f = \emptyset$ $|x| > x \Rightarrow D_f = R^-$

۴) $f(x) = \begin{cases} \dfrac{x^3 - x}{x-1} & x \neq 1 \\ 2 & x = 1 \end{cases}$

$g(x) = x^2 + x$ X برابر

$x^2 + x \geq 0$ $x(x+1) \geq 0$

$-1 \quad 0$

$+ \mid - \mid +$

$D_g = [0, +\infty) \cup [-1, -\infty)$

$\dfrac{x(x^2-1)}{x-1} \geq 0$ $x(x^2-1) \geq 0$

$x \geq 0$ $x^2 \geq 1$

$x \geq \pm 1$ $x - 1 > 0 \to x > 1$

$x > 1$

$D_f = [-1, +\infty) - [1]$

(3

$f(x) \Rightarrow \quad f(x)+g(x) = x^2+x+1$

$+ \quad f(x)-g(x) = x^2-x-1$

$2f(x) = 2x^2+2 \longrightarrow f(x) = x^2+1$

$g(x) = x$

$f^2(x) - g^2(x) \Rightarrow (x^2+1)^2 - x^2$

$x^4 - 1 - x^4 + 2x^2 = \boxed{x^4+1-x^2}$

(4

$x^2 - 4 \geq 0 \quad x^2 \geq 4 \quad \begin{array}{ccc} & -2 & +2 \\ + & - & + \end{array}$

$(x-2)(x+2) \geq 0$

$[-\infty, -2] \cup [2, +\infty) \Rightarrow D_f = D_g \Rightarrow (-\infty, -2) \cup (2, +\infty)$

$D_f \cap D_g$

(5

$f(x) = \dfrac{ax+2}{x^2-mx+n} \qquad x^2-mx+n = 2x^2-3x+5 \Rightarrow m=3, \; n=5$

$ax+2 = x-b$

$am - bn = ?$

$f(g) = \dfrac{x-b}{2x^2-3x+5} \qquad a=1 \quad b=-2$

$ax - x = -b-2$

$x(a-1) = -(b+2) \quad x = \dfrac{-(b+2)}{a-1}$

$1(3) - (\cancel{-2 \times 5}) = 3+10$; -10

$\Rightarrow \boxed{13}$

(6?!

$f(x) = \dfrac{bx+2}{8x+b} \qquad \dfrac{bx+2}{8x+b} = c \Rightarrow bx+2 = 8xc+bc$

$bx - 8xc = bc-2$

$x(b-8c) = bc-2$

$x = \dfrac{bc-2}{b-8c}$

$g(x) = c$

$D_g = \mathbb{R} - \{a\}$

(7

$\dfrac{2g}{f+g} \Rightarrow \quad g = (2,4)(3,6)(-1,-4)(5,0)$

$2f-1 = (-1,3)(2,7)(3,-5)(0,0)$

$\dfrac{2g}{f+g} = \left(-1, \dfrac{8}{3}\right)$

$2g = \{(2,8)(3,12)(-1,-8)(5,0)\}$

$f+g = \{(-1,-2)(2,7)(3,4)\}$

(8

$\boxed{a=-1}$

$\left.\begin{array}{l} 3-2b=-1 \\ -3-2b=1 \end{array}\right\} -2b = 4 \rightarrow \boxed{b=-2}$

$-1+5=4$

$(2,-1) = (2,d) \Rightarrow \boxed{d=-1}$

$a-2b = c$

$-1-2(-2) = c \rightarrow c = 4-1 \Rightarrow \boxed{c=3}$

$d+c = \boxed{-1+5=4}$!

9)

$$f(x) = \sqrt{-(x^2 - x + m)}$$

$$g(x) = [(a,b)] \qquad (x-a)(x-b)$$

$$x^2 - (a+b)x + ab$$

$$-a+b=+1 \Rightarrow \boxed{a+b=1}$$

10)

$x + c$

$$(x+3)^2 = x^2 + 6x + 9 = b \Rightarrow b = 9$$

$$a+b+c = 6+9-3 = \boxed{12}$$

$c = -3$

$a = 6$