نام و نام خانوادگی: تانیا ترک ترابی سری: ۸ گروه: دهم یکشنبه

الف) $f(x)=\sqrt{x|x|}$, $g(x)=x$ $D_f = x|x| \geq 0 \Rightarrow x \geq 0$ $D_g=\mathbb{R}$ برابر نیستند

ب) $f(x)=\frac{(x+3)(x+1)+x+4}{x^2+5x+7}$, $g(x)=1$ $D_f : \frac{(x+3)(x+1)+x+4}{x^2+5x+7} \Rightarrow \frac{x^2+5x+7}{x^2+5x+7}=1$

$(x+3)(x+1)=x^2+x+3x+3 \Rightarrow x^2+4x+3+x+4=x^2+5x+7$ $\overset{\mathbb{R}}{D_f}=\overset{\mathbb{R}}{D_g}$ برابر هستند

ج) $f(x)=\frac{4\sin^2 x-6\sin x}{2\sin x-3}$, $g(x)=2\sin x$ $D_f=2\sin x-3\neq 0 \Rightarrow 2\sin x\neq 3 \Rightarrow \sin x\neq \frac{3}{2} \Rightarrow$ ۱

$-1\leq \sin x\leq 1 \Rightarrow D_f=\mathbb{R}$ $D_g=\mathbb{R}$ $D_f=D_g$ برابر هستند

د) $f(x)=\frac{x}{|x|}$, $g(x)=\frac{|x|}{x}$ $D_f=|x|\neq 0 \Rightarrow x\neq 0 \Rightarrow D_f=\mathbb{R}-\{0\}$ $D_g=x\neq 0\Rightarrow D_g=\mathbb{R}-\{0\}$ برابر هستند

الف) $f(x)=\left[\frac{x^2}{x^2+1}\right]$, $g(x)=\left[x-[x]\right]$ $D_f=x^2+1\neq 0 \Rightarrow x^2\neq -1 \Rightarrow D_f=\mathbb{R}$ برابر هستند

$D_g=\mathbb{R}$ $D_f=D_g$

ب) $f(x)=\frac{1}{[2x]}$, $g(x)=\frac{1}{2[x]}$ $D_f=[2x]\neq 0 \Rightarrow 0\leq 2x<1 \Rightarrow 0\leq x<\frac{1}{2} \Rightarrow D_f=\mathbb{R}-[0,\frac{1}{2})$

$D_g=2[x]\neq 0 \Rightarrow [x]\neq 0 \Rightarrow 0\leq x<1 \Rightarrow D_g=\mathbb{R}-[0,1)$ $D_f\neq D_g$ برابر نیستند ۲

ج) $f(x)=\frac{1}{\sqrt{x-|x|}}$, $g(x)=\frac{1}{\sqrt{|x|-x}}$ $D_f=x-|x|>0 \Rightarrow x>|x| \Rightarrow D_f=\varnothing$ $D_g=|x|-x>0\Rightarrow |x|>x$ برابر نیستند

$D_g=(-\infty,0)$ $D_f\neq D_g$

د) $f(x)=\begin{cases}\frac{x^3-x}{x-1} ; & x\neq 1\\ 2 ; & x=1\end{cases}$, $g(x)=x^2+x$ $D_f = ①\ x-1\neq 0 \Rightarrow x\neq 1$ ✓ $x=1$ اگر $\Rightarrow g(1)=2=f(1)$ برابر هستند

$f(x)+g(x)=x^2+x+1$
$f(x)-g(x)=x^2-x+1$

$\begin{cases}(f(x)+g(x))+(f(x)-g(x))=(x^2+x+1)+(x^2-x+1) \Rightarrow \\ 2f(x)=2x^2+2 \Rightarrow \boxed{f(x)=x^2+1}\end{cases}$

$(f(x)+g(x))-(f(x)-g(x))=(x^2+x+1)-(x^2-x+1) \Rightarrow 2g(x)=2x \Rightarrow \boxed{g(x)=x}$

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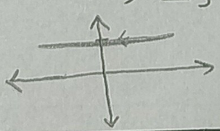
$f(x)=x-\sqrt{x^2-4}$, $g(x)=x+\sqrt{x^2-4}$ $f(x)\times g(x)=(\underbrace{x-\sqrt{x^2-4}}_{(a-b)})\times(\underbrace{x+\sqrt{x^2-4}}_{(a+b)})=a^2-b^2$

$\Rightarrow f(x)\times g(x)=x^2-(\sqrt{x^2-4})^2 \Rightarrow f(x)\times g(x)=x^2-(x^2-4) \Rightarrow f(x)\times g(x)=x^2-x^2+4$ ۴

$\Rightarrow f(x)\times g(x)=4$



$\frac{x-b}{2x^2-3x-5}=\frac{ax+2}{x^2-mx+n} \Rightarrow x^2-mx+n=k(2x^2-3x-5) \Rightarrow 1=2k \Rightarrow k=\frac{1}{2}$

$ax+2=k(x-b) \Rightarrow ax+2=\frac{1}{2}(x-b) \Rightarrow ax+2=\frac{1}{2}x-\frac{1}{2}b \Rightarrow a=\frac{1}{2}$ ۵

$2=-\frac{1}{2}b \Rightarrow b=-4$, $x^2-mx+n=k(2x^2-3x-5) \Rightarrow x^2-mx+n=\frac{1}{2}(2x^2-3x-5)$

$\Rightarrow x^2-mx+n=x^2-\frac{3}{2}x-\frac{5}{2} \Rightarrow -m=-\frac{3}{2} \Rightarrow m=\frac{3}{2}$, $n=-\frac{5}{2}$, $a=\frac{1}{2}$, $b=-4$

$am-bn=(\frac{1}{2}\times\frac{3}{2})-(-4\times-\frac{5}{2})=-\frac{37}{4}$

$x \neq -\frac{b}{8}$

$\frac{bx+2}{8x+b} = c \Rightarrow 8x+b \neq 0 \Rightarrow \frac{b}{8} = c \Rightarrow b = 8c$

$\frac{2}{b} = c \Rightarrow 2 = bc \Rightarrow 2 = 8c^2 \Rightarrow c^2 = \frac{2}{8} = \frac{1}{4} \Rightarrow$ ① $c = \frac{1}{2}$ یا ② $c = -\frac{1}{2}$ ۶

① $b = 8 \times \frac{1}{2} = 4$ ② $b = 8 \times (-\frac{1}{2}) = -4 \Rightarrow \frac{ab}{c} \Rightarrow \frac{a \times 4}{\frac{1}{2}} = a \times 8$ ①

② $\frac{a \times -4}{-\frac{1}{2}} = a \times 8$

$g = \{(2,4),(3,6),(-1,-4),(5,0)\} \Rightarrow 2g = \{(2,8),(3,12),(-1,-8),(5,0)\}$

$2f-1 = \{(-1,3),(2,7),(3,-5),(0,0)\} \Rightarrow f = \{(-1,2)(2,4),(3,-2),(0,\frac{1}{2})\}$

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$f = \{(2,\underset{-1}{a}),(-3,3a-2b),(c,5),(2,\underset{-1}{d})\} = g = \{(2,-1),(-3,1),(a-3b,5)\}$

$a = -1,\ b = -2$

$3\underset{-1}{a} - 2b = 1 \Rightarrow -3 - 2b = 1 \Rightarrow -2b = 1+3 \Rightarrow -2b = 4 \Rightarrow b = -2$ $\quad d = -1$

$a - 3b \Rightarrow -1 - 3(-2) = 5 \quad \underbrace{a-3b}_{5} = c \Rightarrow c = 5 \quad d + c = -1 + 5 = 4$ ۸

$f(x) = \sqrt{\underbrace{-x^2+x-m}_{max}} \Rightarrow \Delta = b^2 - 4ac = (1)^2 - 4(-1)(-m) = 1 - 4m \xRightarrow{\Delta = 0}$

$1 - 4m = 0 \Rightarrow 4m = 1 \Rightarrow m = \frac{1}{4} \Rightarrow -x^2 + x - \frac{1}{4} = -(x^2 - x + \frac{1}{4}) =$ ۹

$-(x-\frac{1}{2})^2 \Rightarrow f(x) = \sqrt{-(x-\frac{1}{2})^2} \Rightarrow -(x-\frac{1}{2})^2 \geq 0 \Rightarrow (x-\frac{1}{2})^2 = 0 \Rightarrow x - \frac{1}{2} = 0 \Rightarrow x = \frac{1}{2}$

$f(\frac{1}{2}) = \sqrt{-(\frac{1}{2}-\frac{1}{2})^2} = 0 \Rightarrow f = \{(\frac{1}{2},0)\} \quad a = \frac{1}{2},\ b = 0 \quad a + b = \frac{1}{2} + 0 = \frac{1}{2}$

$\frac{2x+a}{x^2+2x+b} = \frac{2}{x-c} \Rightarrow x \neq c$ ۱۰