نام و نام خانوادگی: تانیا ترک ترابی   سری: ۸   گروه: دهم یکشنبه

۱۵,۵

الف) $f(x)=\sqrt{x|x|}\ ,\ g(x)=x$ $\quad D_f = x|x|\geq 0 \Rightarrow x\geq 0 \quad D_g=\mathbb{R}$ برابر نیستند

ب) $f(x)=\frac{(x+3)(x+1)+x+4}{x^2+5x+7}\ ,\ g(x)=1 \quad D_f: \frac{(x+3)(x+1)+x+4}{x^2+5x+7} = \frac{x^2+5x+7}{x^2+5x+7}=1$

$(x+3)(x+1) = x^2+x+3x+3 = x^2+4x+3+x+4 = x^2+5x+7 \quad \overset{\mathbb{R}}{D_f} = \overset{\mathbb{R}}{D_g}$ برابر هستند

ج) $f(x)=\frac{4\sin^2x-6\sin x}{2\sin x-3}\ ,\ g(x)=2\sin x \quad D_f = 2\sin x-3\neq 0 \Rightarrow 2\sin x\neq 3 \Rightarrow \sin x\neq \frac{3}{2} \Rightarrow$ (۲) ۱

$-1\leq \sin x\leq 1 \Rightarrow D_f=\mathbb{R} \quad D_g=\mathbb{R} \quad D_f=D_g$ برابر هستند

د) $f(x)=\frac{x}{|x|}\ ,\ g(x)=\frac{|x|}{x} \quad D_f = |x|\neq 0 \Rightarrow x\neq 0 \Rightarrow D_f=\mathbb{R}-\{0\} \quad D_g = x\neq 0 \Rightarrow D_g = \mathbb{R}-\{0\}$ برابر هستند

الف) $f(x)=\left[\frac{x^2}{x^2+1}\right]\ ,\ g(x)=\left[x-[x]\right] \quad D_f = x^2+1\neq 0 \Rightarrow x^2\neq -1 \Rightarrow D_f=\mathbb{R}$

$D_g=\mathbb{R} \quad D_f=D_g$ برابر هستند

ب) $f(x)=\frac{1}{[2x]}\ ,\ g(x)=\frac{1}{2[x]} \quad D_f=[2x]\neq 0 \Rightarrow 0\leq 2x<1 \Rightarrow 0\leq x<\frac{1}{2} \Rightarrow D_f=\mathbb{R}-[0,\frac{1}{2})$ (۲)

$D_g = 2[x]\neq 0 \Rightarrow [x]\neq 0 \Rightarrow 0\leq x<1 \Rightarrow D_g=\mathbb{R}-[0,1) \quad D_f\neq D_g$ برابر نیستند ۲

ج) $f(x)=\frac{1}{\sqrt{x-|x|}}\ ,\ g(x)=\frac{1}{\sqrt{|x|-x}} \quad D_f = x-|x|>0 \Rightarrow x>|x| \Rightarrow D_f=\varnothing \quad D_g=|x|-x>0 \Rightarrow |x|>x$

$D_g=(-\infty,0) \quad D_f\neq D_g$ برابر نیستند

د) $f(x)=\begin{cases}\frac{x^3-x}{x-1}\ ; & x\neq 1\\ 2\ ; & x=1\end{cases}\ ,\ g(x)=x^2+x \quad D_f = ①\ x-1\neq 0 \Rightarrow x\neq 1$ اگر $x=1 \Rightarrow g(x)=2=f(x)$ برابر هستند

$f(x)+g(x)=x^2+x+1$

$f(x)-g(x)=x^2-x+1$

$\begin{cases}(f(x)+g(x))+(f(x)-g(x)) = (x^2+x+1)+(x^2-x+1) \Rightarrow \\ 2f(x)=2x^2+2 \Rightarrow \boxed{f(x)=x^2+1}\end{cases}$ (۱,۵)

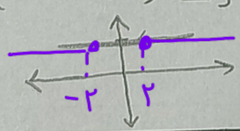
$(f(x)+g(x))-(f(x)-g(x)) = (x^2+x+1)-(x^2-x+1) \Rightarrow 2g(x)=2x \Rightarrow \boxed{g(x)=x}$ ۳

$f^2(x)-g^2(x) = (x^2+1)^2-x^2 = x^4+x^2+1$

$f(x)=x-\sqrt{x^2-4}\ ,\ g(x)=x+\sqrt{x^2-4} \quad f(x)\times g(x) = \underbrace{(x-\sqrt{x^2-4})}_{(a-b)}\times\underbrace{(x+\sqrt{x^2-4})}_{(a+b)} = a^2-b^2$ ۴

$\Rightarrow f(x)\times g(x) = x^2-(\sqrt{x^2-4})^2 \Rightarrow f(x)\times g(x) = x^2-(x^2-4) \Rightarrow f(x)\times g(x) = x^2-x^2+4$ (۱)

$\Rightarrow f(x)\times g(x)=4$



$D_{f\times g} = x^2\geq 4 \to x\geq 2$ یا $x\leq -2$

$\frac{x-b}{2x^2-3x-5} = \frac{ax+2}{x^2-mx+n} \Rightarrow x^2-mx+n = k(2x^2-3x-5) \Rightarrow 1=2k \Rightarrow k=\frac{1}{2}$

(۲)

$ax+2 = k(x-b) \Rightarrow ax+2 = \frac{1}{2}(x-b) \Rightarrow ax+2 = \frac{1}{2}x-\frac{1}{2}b \Rightarrow a=\frac{1}{2}$ ۵

$2=-\frac{1}{2}b \Rightarrow b=-4 \quad ,\ x^2-mx+n = k(2x^2-3x-5) \Rightarrow x^2-mx+n = \frac{1}{2}(2x^2-3x-5)$

$\Rightarrow x^2-mx+n = x^2-\frac{3}{2}x-\frac{5}{2} \Rightarrow -m=-\frac{3}{2} \Rightarrow m=\frac{3}{2}\ ,\ n=-\frac{5}{2}\ ,\ a=\frac{1}{2}\ ,\ b=-4$

$am-bn = (\frac{1}{2}\times\frac{3}{2})-(-4\times-\frac{5}{2}) = -\frac{37}{4}$

$x \neq -\frac{b}{8}$

$\frac{bx+2}{8x+b} = c \Rightarrow 8x+b \neq 0 \Rightarrow \frac{b}{8} = c \Rightarrow b = 8c$ ۱/۵

$\frac{2}{b} = c \Rightarrow 2 = bc \Rightarrow 2 = 8c^2 \Rightarrow c^2 = \frac{2}{8} = \frac{1}{4} \Rightarrow$ ① $c = \frac{1}{2}$ یا ② $c = -\frac{1}{2}$ ۶

① $b = 8 \times \frac{1}{2} = 4$ ② $b = 8 \times (-\frac{1}{2}) = -4 \Rightarrow \frac{ab}{c} \Rightarrow \frac{a \times 4}{\frac{1}{2}} = a \times 8$ ①

② $\frac{a \times -4}{-\frac{1}{2}} = a \times 8$ $c = \frac{1}{2} \to b = 4 \to a = -\frac{1}{2} \to \frac{ab}{c} = -4$

$c = -\frac{1}{2} \to b = -4 \to a = \frac{1}{2} \to \frac{ab}{c} = 4$

$g = \{(2,4),(3,6),(-1,-4),(5,0)\} \Rightarrow 2g = \{(2,8),(3,12),(-1,-8),(5,0)\}$

$2f-1 = \{(-1,3),(2,7),(3,-5),(0,0)\} \Rightarrow f = \{(-1,2)(2,4),(3,-2),(0,\frac{1}{2})\}$ ۱/۵

$\frac{2g}{f+g} = \{(-1, \frac{8}{-2})(3, \frac{12}{4})(2, \frac{8}{8})\}$ ۷

$f = \{(2,a),(-3,3a-2b),(c,5),(2,d)\} = g = \{(2,-1),(-3,1),(a-3b,5)\}$
($a = -1$, $d = -1$)

$a = -1,\ b = -2$

$3a - 2b = 1 \Rightarrow -3 - 2b = 1 \Rightarrow -2b = 1+3 \Rightarrow -2b = 4 \Rightarrow b = -2$ $d = -1$ ۸

$a - 3b \Rightarrow -1 - 3(-2) = 5$ $\underbrace{a-3b}_{5} = c \Rightarrow c = 5$ $d + c = -1 + 5 = 4$ ۲

$f(x) = \sqrt{\underbrace{-x^2+x-m}_{max}} \Rightarrow \Delta = b^2 - 4ac = (1)^2 - 4(-1)(-m) = 1 - 4m \xRightarrow{\Delta = 0}$

$1 - 4m = 0 \Rightarrow 4m = 1 \Rightarrow m = \frac{1}{4} \Rightarrow -x^2 + x - \frac{1}{4} = -(x^2 - x + \frac{1}{4}) =$ ۲ ۹

$-(x-\frac{1}{2})^2 \Rightarrow f(x) = \sqrt{-(x-\frac{1}{2})^2} \Rightarrow -(x-\frac{1}{2})^2 \geq 0 \Rightarrow (x-\frac{1}{2})^2 = 0 \Rightarrow x - \frac{1}{2} = 0 \Rightarrow x = \frac{1}{2}$

$f(\frac{1}{2}) = \sqrt{-(\frac{1}{2}-\frac{1}{2})^2} = 0 \Rightarrow f = \{(\frac{1}{2}, 0)\}$ $a = \frac{1}{2},\ b = 0$ $a + b = \frac{1}{2} + 0 = \frac{1}{2}$

$\frac{2x+a}{x^2+4x+b} = \frac{2}{x-c} \Rightarrow x \neq c$ $a + b + c = 12$ ۰ ۱۰

$\hookrightarrow \Delta = 0 \to b = 4$

$\frac{2}{x+2} = \frac{2x+a}{(x+2)^2} \to a = 4$

$D_f = D_g = \mathbb{R} - \{-2\} \to c = -2$