

1. الف) $f(x) = \sqrt{x|x|}$, $g(x) = x$ ✓
 $D_f = \mathbb{R}^+ \cup \{0\}$ $x|x| \geq 0$ $D_g = \mathbb{R}$

ب) $f(x) = \frac{(x^2+1)(x+1)+x+1}{(x^2+\omega x+1)}$, $g(x) = 1$ ✓
 $D_g = \mathbb{R}$ $D_f = \mathbb{R}$ $x^2+\omega x+1$

ج) $f(x) = \frac{r \sin x - 4 \sin x}{r \sin x - r}$, $g(x) = r \sin x$ ✓
 $D_g = \mathbb{R}$ $r \sin x (r \sin x - r) = r \sin x$

$r \sin x - r \neq 0$ $r \sin x + r$ $\sin x \neq \frac{r}{r}$ $D_f = \mathbb{R}$

د) $f(x) = \frac{x}{|x|}$, $g(x) = \frac{|x|}{x}$ ✓
 $x > 0 \rightarrow 1$, $x < 0 \rightarrow -1$, $x = 0 \rightarrow$ نقص
 $|x| \neq 0 \rightarrow x \neq 0$ $x \neq 0$ $D_f = \mathbb{R} - \{0\}$ $D_g = \mathbb{R} - \{0\}$ نقص

2. الف) $f(x) = \left[\frac{x}{x+1} \right]$, $g(x) = [x - [x]]$ ✓
برای $x \in \mathbb{Z}$ و $x \in \mathbb{R}$ بررسی می‌کنیم

ب) $f(x) = \frac{1}{[x]}$, $g(x) = \frac{1}{r[x]}$ ✗
 $D_f = \mathbb{R} - [0, 1)$ $D_g = \mathbb{R} - [0, 1)$ نقص

ج) $f(x) = \frac{1}{\sqrt{x-|x|}}$, $g(x) = \frac{1}{\sqrt{|x|-x}}$ ✗
برای $x \neq 0$ بررسی می‌کنیم

$|x| \neq 0 \rightarrow x \neq 0$ $|x| > 0$ $x > |x|$ $\sqrt{|x|-x} \neq 0$ $(|x|-x) > 0$ $|x| > x$
 $D_f = \emptyset$ $D_g = (-\infty, 0)$

د) $f(x) = \begin{cases} \frac{x^2-x}{x-1} & ; x \neq 1 \\ r & ; x = 1 \end{cases}$, $g(x) = \frac{x^2+x}{x+1}$ ✓
 $D_f = \mathbb{R}$ $D_g = \mathbb{R}$

$\frac{x(x^2-1)}{x-1} = \frac{x(x+1)(x-1)}{x-1} = x(x+1) = x^2+x$

3. $f(x) + g(x) = x^2 + x + 1$ $x^2 + 1 + g(x) = x^2 + x + 1$ $g(x) = x^2 + x + 1 - x^2 - 1 = x$ ✓
 $f(x) - g(x) = x^2 - x + 1$ $f(x) = x^2 + x$ $f(x) = x^2 + 1$

$f'(x) - g'(x) = x^2 + 2x + 1 - x^2 = 2x + 1$

$$f(x) = x - \sqrt{x^2 - c} \quad f \times g = (x - \sqrt{x^2 - c})(x + \sqrt{x^2 - c}) = x^2 - x^2 + c = c$$

$$g(x) = x + \sqrt{x^2 - c}$$

$$D_f = (-\infty, -r] \cup [r, +\infty) \quad D_g = (-\infty, -r] \cup [r, +\infty)$$

$$f(x) = \frac{ax+r}{x^2-mx+n} \quad x^2-mx+n = (x+1)(x-\frac{\omega}{r}) = x^2 - \frac{\omega}{r}x - \frac{\omega}{r} \quad m = \frac{r}{r} \quad n = -\frac{\omega}{r}$$

$$g(x) = \frac{x-b}{rx^2-rx-\omega} \rightarrow \frac{x-b}{(rx-\omega)(x+1)} \quad \frac{1}{r} \rightarrow rx + r = x - b \rightarrow a = \frac{1}{r} \quad b = -r \quad D_g = \mathbb{R} - \{\frac{\omega}{r}, -1\} \quad am - bn = \frac{r}{r} - \frac{\omega}{r} = 1 - \frac{\omega}{r}$$

$$f(x) = \frac{bx+r}{ax+b} \rightarrow bx+r = \lambda(ax+b) \rightarrow b = \lambda a \quad r = \lambda b \quad \lambda a + b = 0$$

$$g(x) = c \quad D_g = \mathbb{R} - \{a\} \quad \frac{b}{a} = \frac{r}{b} \Rightarrow b^2 = ar \quad b = \pm \sqrt{ar}$$

$$f = \{(-1, r)(r, v)(r, -\omega)(\omega, 0)\} \quad f = \{(-1, r)(r, r)(r, -r)(\omega, \frac{r}{r})\}$$

$$g = \{(r, r)(r, r)(-1, -r)(\omega, 0)\} \quad \frac{rg}{f+g} = \{(r, \frac{r}{r})(r, r)(-1, \frac{r}{r})\}$$

$$f = \{(r, a)(-r, ra-rb)(c, \omega)(r, d)\} \quad a = -1 \quad d+c = -1+\omega = r$$

$$ra-rb = 1 \quad -r-rb = 1 \quad -rb = r \quad b = -r$$

$$g = \{(r, -1)(-r, 1)(a-rb, \omega)\} \quad -1+r = \omega \quad c = \omega \quad d = -1$$

$$f(x) = \frac{\sqrt{-x^2+x-m}}{x^2+ax+b} \quad -x^2+x-m \geq 0 \quad x^2+ax+b \geq 0 \quad \Delta = 0, b = x^2+x-m$$

$$g(x) = \{(a, b)\} \quad (a, b) \rightarrow y=0 \Rightarrow b=0$$

$$x^2+ax-\frac{1}{r} \rightarrow -a^2+a-\frac{1}{r} = 0 \quad a = \frac{1}{r}$$

$$f(x) = \frac{rx+a}{x^2+ax+b} \quad \frac{rx+a}{x^2+ax+b} = \frac{r}{x-c} \quad a-rc = 1r \quad c = r \quad a = r$$

$$g(x) = \frac{r}{x-c} \quad rx^2+rx+rb = rx^2+ax-rxc-ac$$

$$rx^2+rx+rb = rx^2+(a-rc)x-ac$$

$$a-rc = 1r \quad a = 1r+rc$$

$$\frac{rb}{r} = \frac{-ac}{r} \rightarrow b = \frac{-ac}{r} \quad a+b+c = (r-r+r) = r$$