(1)

$$y=\frac{x+4}{4x^3-20x^2+31x-15}$$

جمع ضرایب صفر

$$\Rightarrow \frac{x+4}{(x-1)(4x^2-16x+15)}$$

$$4x^3-20x^2+31x-15 \;\Big|\; x-1 \quad\Rightarrow\quad 4x^2-16x+15$$

$$\begin{array}{l} -4x^3 + 4x^2 \\ \hline -16x^2+31x \\ +16x^2-16x \\ \hline 15x-15 \\ -15x+15 \\ \hline 0 \end{array}$$

$(+1)$; $4\times 15=60 \Rightarrow (x-10)(x-6)$; $x=\frac{+10}{4}$; $x=\frac{+6}{4}$

$$D_f=\mathbb{R}-\left\{+1, +\frac{6}{4}, +\frac{10}{4}\right\}$$

$\frac{6}{4}=1.5=\frac{3}{2}$; $\frac{10}{4}=2.5$

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(1)

$$y=\frac{x+3}{3x^3-x^2-8x-4}=\frac{x+3}{(x+1)(3x^2-4x-4)}$$

$-5 \quad -5$

$$3x^3-x^2-8x-4 \;\Big|\; x+1 \quad\Rightarrow\quad 3x^2-4x-4$$

$$\begin{array}{l} -3x^3 - 3x^2 \\ \hline -4x^2-8x-4 \\ +4x^2+4x \\ \hline -4x-4 \\ +4x+4 \\ \hline 0 \end{array}$$

(-1) ; $3\times(-4)=-12$; $(x-6)(x+2)$; $x=\frac{+6}{3}$; $x=-\frac{2}{3}$

$$D_f=\mathbb{R}-\left\{-1, +\frac{6}{3}, -\frac{2}{3}\right\}$$

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الف) $y=\dfrac{x+1}{x-\sqrt{3-2x}}$; $3-2x\geq 0 \Rightarrow x\leq \frac{3}{2}$

$x-\sqrt{3-2x}\neq 0$

$x\neq\sqrt{3-2x}$

$x^2\neq 3-2x$

$x^2+2x-3=0$; $(x-1)(x+3)$; $x=1$; $x=-3$

$$\Rightarrow D_f=\left(-\infty, \frac{3}{2}\right]-\{1,-3\}$$

ب) $y=\dfrac{x+2}{x-\sqrt{3x-2}}$; $3x-2\geq 0$; $x\geq\frac{2}{3}$

$x\neq\sqrt{3x-2}$

$x^2\neq 3x-2$; $x^2-3x+2\neq 0$

$(x-1)(x-2)\rightarrow x\neq 1$; $x\neq 2$

$$\Rightarrow D_f=\left[\frac{2}{3}, +\infty\right)-\{1,2\}$$

$x^2 \neq 3x - 2$ $\quad (x-1)(x-2) \to x \neq 1,\ x \neq 2$

۳-

الف) $y = \dfrac{\sin x}{2\cos x - 1}$ $\qquad 2\cos x - 1 \neq 0$, $\cos x \neq \dfrac{1}{2}$, $x = 2k\pi \pm \dfrac{\pi}{3}$, $\qquad D_f = \mathbb{R} - \left\{\dfrac{\pi}{3} + 2k\pi,\ 2k\pi - \dfrac{\pi}{3}\right\}$

ب) $y = \dfrac{\cos x + 1}{2\sin x + 1}$ $\qquad \sin x \neq -\dfrac{1}{2}$ $\qquad D_f = \mathbb{R} - \left\{2k\pi + \dfrac{7\pi}{6},\ 2k\pi + \dfrac{11\pi}{6}\right\}$

ج) $y = \dfrac{\tan x + 2}{\cot x - 1}$ $\qquad \cot x \neq 1 \to \cos x \neq \sin x$ $\qquad D_f = \mathbb{R} - \left\{k\pi + \dfrac{\pi}{4}\right\}$

$D_f = \mathbb{R} - \left\{k\pi,\ k\pi + \dfrac{\pi}{2},\ k\pi + \dfrac{\pi}{4}\right\}$

د) $y = \dfrac{\sin x + 1}{4\sin^2 x - 3}$ $\qquad 4\sin^2 x - 3 \neq 0$, $\sin^2 x \neq \dfrac{3}{4}$, $\sin x \neq \pm\dfrac{\sqrt{3}}{2}$ $\qquad D_f = \mathbb{R} - \left\{\dfrac{\pi}{3} + 2k\pi,\ \dfrac{2\pi}{3} + 2k\pi,\ -\dfrac{\pi}{3} + 2k\pi,\ -\dfrac{2\pi}{3} + 2k\pi\right\}$

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الف) $x^2 - 5x + 6 \geq 0$ $\quad (x-2)(x-3) \geq 0$ $\quad (+2)\ (+3)$ $\quad$ $\begin{array}{cccc} & +2 & & +3 \\ + & | & - & | & + \end{array}$ $\quad D_f = (-\infty, 2] \cup [3, +\infty)$

ب) $x^2 - 6x + 5 \leq 0$ $\quad (x-1)(x-5) \leq 0$ $\quad (+1)\ (+5)$ $\quad$ $\begin{array}{ccccc} & 1 & & 5 & \\ + & | & - & | & + \end{array}$ $\quad D_f = [1, 5]$

ج) $x^2 + 6x + 5 \geq 0$ $\quad (x+1)(x+5) \geq 0$ $\quad (-1)\ (-5)$ $\quad$ $\begin{array}{ccccc} & -5 & & -1 & \\ + & | & - & | & + \end{array}$ $\quad D_f = (-\infty, -5] \cup [-1, +\infty)$

د) $x^2 - 7x + 6 \leq 0$ $\quad (x-1)(x-6) \leq 0$ $\quad (+1)\ (+6)$ $\quad$ $\begin{array}{ccccc} & 1 & & 6 & \\ + & | & - & | & + \end{array}$ $\quad D_f\ [1, 6]$

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الف) $\dfrac{x^2 - 3x + 2}{x - 5} \leq 0 \to (x-1)(x-2)$ $\quad (+1)\ (+2)$, $x - 5 \to (+5)$ $\quad$ $\begin{array}{ccccccc} & 1 & & 2 & & 5 & \\ - & \phi & + & \phi & - & \phi & + \end{array}$ $\quad D_f\ (-\infty, 1] \cup [2, 5)$

ب) $\dfrac{x^2 - 1}{x^2 - 6x + 5} \geq 0$ $\quad \to x^2 = 1$, $x = \pm 1$ $\quad x^2 - 6x + 5 \to (x-1)(x-5)$ $\quad (+1)\ (+5)$ $\quad$ $\begin{array}{ccccccc} & -1 & & 1^* & & 5 & \\ + & \phi & - & \phi & - & \phi & + \end{array}$ $\quad D_f = (-\infty, -1] \cup (5, +\infty)$

$\sqrt{\frac{x^2-4x+3}{x^3-1}}\geq 0 \qquad x^3\neq 1 \qquad x\neq 1 \qquad \frac{(x-1)(x-3)}{x^3-1}\geq 0$ -۶

	$+1$		$+3$	
$+$	$\emptyset$	$-$	$\emptyset$	$+$

$D_f=(-\infty,+1)\cup[+3,+\infty)$

ب) $y=\sqrt[3]{\frac{x^2-4x+3}{x^2-1}}$ $\qquad x^2\neq 1 \qquad x\neq\pm 1 \qquad D_f=\mathbb{R}-\{1,2\}$

الف) $y=\log(x^2-3x)$ -۷

$x^2-3x>0$

$x(x-3)>0$

	0		$+3$	
$+$	$\emptyset$	$-$	$\emptyset$	$+$

$\Rightarrow D_f=(-\infty,0)\cup(+3,+\infty)$

ب) $y=\log_{|x|-2}^{16-x^2}$

$16-x^2>0 \qquad |x|-2>0 \qquad |x|\neq 3$

$16>x^2 \Rightarrow -4<x<4 \qquad |x|>2 \Rightarrow x>2,\ x<-2 \qquad x\neq\pm 3$

$D_f=(-4,-2)\cup(+2,+4)-\{\pm 3\}$

ج) $y=\log_{10-x}\frac{x^2-7x+12}{x-3} \rightarrow (x-3)(x-4)$

	3		4	
$-$	$\emptyset$	$-$	$\emptyset$	$+$

$\frac{(x-3)(x-4)}{x-3}>0 \qquad D_f=(4,10)-\{9\}$

$[4,+\infty)$

$10-x \rightarrow 10>x$

$x\neq 9 \qquad x\neq 3$

د) $y=\sqrt{x^2-11x+28}+\log_x\sqrt{36-x^2}$ $\qquad (x-4)(x-7)\geq 0$

	4		7	
$+$		$-$		$+$

$D_f=[$

$\sqrt{36-x^2}>0 \Rightarrow 36-x^2>0 \qquad 36>x^2$

$-6<x<6$

$36-x^2\geq 0 \Rightarrow -6\leq x\leq 6 \qquad x\neq 1 \qquad D_f=(0,4]-\{1\}$

$36 \qquad x>0$

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$y = \log_{\ } |x|-2$ $\quad x \neq \pm 2$

$16 > x^2 \Rightarrow -4 < x < 4 \quad |x| > 2 \Rightarrow x > 2,\ x < -2$

ج) $y = \log_{10-x} \frac{x^2-7x+12}{x-3} \rightarrow (x-3)(x-4) \quad \frac{(x-3)(x-4)}{x-3} > 0 \quad D_f = (4,10) - \{9\}$

$10-x \rightarrow 10 > x \quad x \neq 9 \quad x \neq 3 \quad [4,+\infty)$

د) $y = \sqrt{x^2-11x+28} + \log_x \sqrt{36-x^2} \quad (x-4)(x-7) \geq 0$

	4		7	
+	|	−	|	+

$D_f = [$

$\sqrt{36-x^2} > 0 \Rightarrow 36-x^2 > 0 \quad 36 > x^2 \quad -6 < x < 6 \quad D_f = (0,4] - \{1\}$

$36-x^2 \geq 0 \Rightarrow -6 \leq x \leq 6 \quad x \neq 1$

$x > 0$

$y = \frac{x^2-x}{x^2+x} \quad x(x-1) \rightarrow 0,\ +1 \quad x(x+1) \rightarrow 0,\ -1$

	−1		0		+1	
$x(x-1)$	+	+	0	−	0	+
$x(x+1)$	+	−		+		+
$f(x)$	+	−		−	0	+

	−1		0		+1	
+	|	−	|	−	|	+

$\sqrt{\frac{x-f(x)}{f(x)}} \Rightarrow \frac{x-f(x)}{f(x)} \geq 0 \rightarrow$

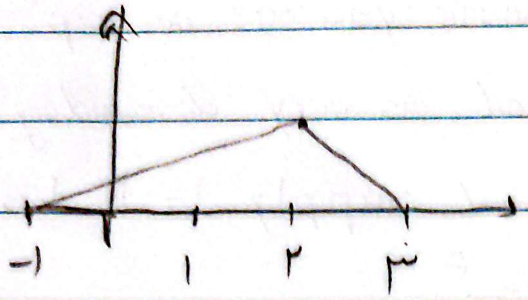
$x = f(x)$ نقاط تلاقی $\rightarrow (-1,3)$

$\Rightarrow D_f = (-2,-1] , (+2,3]$

$f(x) \neq 0$

$x \neq \pm 2$

	−2		−1		+2		+3	
−	|	+	0	−	|	+	0	−



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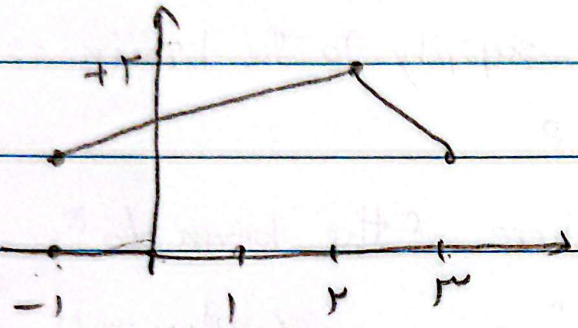

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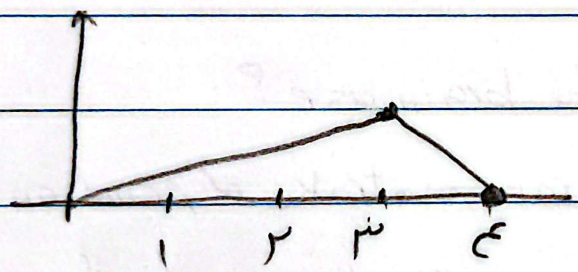

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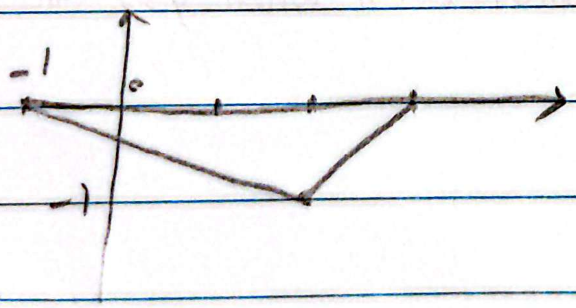

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