تابع و خواص آن‌ها

سنا افشین

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$$y=\frac{x+2}{4x^3-20x^2+31x-15} \qquad x=1 \rightarrow (x-1)$$

$$4x^3-20x^2+31x-15 \div x-1 = 4x^2-16x+15$$

$\rightarrow (2x-5)(2x-3) \rightarrow$ ریشه‌ها $\frac{3}{2}, \frac{5}{2}, 1$

$$\begin{array}{ccccccc} & 1 & & \frac{3}{2} & & \frac{5}{2} & \\ - & 0 & + & | & - & | & + \end{array} \qquad D_f=[1,\tfrac{3}{2})\cup(\tfrac{5}{2},+\infty)$$

$$y=\frac{x+3}{3x^3-x^2-8x-4} \qquad x=-1 \rightarrow (x+1)$$

$$3x^3-x^2-8x-4 \div x+1 = 3x^2-4x-4$$

$(3x+2)(x-2) \rightarrow$ ریشه‌ها $= \frac{2}{3}, -\frac{2}{3}, -1$

$$\begin{array}{ccccccc} & -\frac{2}{3} & & -1 & & \frac{2}{3} & \\ - & | & + & 0 & - & | & + \end{array}$$

$$D_f=(-\tfrac{2}{3},-1]\cup(\tfrac{2}{3},+\infty)$$

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$$y=\frac{x+1}{x-\sqrt{3-2x}} \neq 0 \rightarrow (x\neq\sqrt{3-2x})^2 \rightarrow x^2\neq 3-2x$$

$$\rightarrow x^2+2x\neq 3 \rightarrow x(x+1)\neq 2 \qquad D_f=\mathbb{R}-\{-1,0\}$$

$$y=\frac{x+2}{x-\sqrt{3x-2}} \neq 0 \rightarrow (x\neq\sqrt{3x-2})^2 \rightarrow x^2\neq 3x-2$$

$$x^2-3x\neq -2 \rightarrow x(x-3)\neq -2 \qquad D_f=\mathbb{R}-\{0,3\}$$

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$$y=\frac{\sin x}{2\cos x-1} \neq 0 \rightarrow 2\cos x\neq 1 \rightarrow \cos x\neq \tfrac{1}{2}$$

$$D_f=\mathbb{R}-\{2k\pi+\tfrac{2\pi}{3},\ 2k\pi-\tfrac{\pi}{3}\}$$

$$y = \frac{\cos x + 1}{2\sin x + 1 \neq 0} \quad 2\sin x \neq -1 \to \sin x \neq -\frac{1}{2}$$

$$D_f = \mathbb{R} - \{2k\pi + \pi + \frac{\pi}{6}, 2k\pi - \frac{\pi}{6}\}$$

$$y = \frac{\tan x + 2}{\cot x - 1 \neq 0} \to \cot x \neq 1 \qquad \cot = \frac{\cos}{\sin}$$

$$D_f = \{2k\pi, 2k\pi + \frac{\pi}{2}\} \cup \{2k\pi + \pi, 2k\pi + \frac{3\pi}{2}\}$$

$$y = \frac{\sin x + 1}{4\sin^2 x - 3 \neq 0} \to 4\sin^2 x \neq 3 \to \sin^2 x \neq \frac{3}{4}$$

$$\to \sin x \neq \pm\sqrt{\frac{3}{4}}$$

$$D_f = \mathbb{R} - \{2k\pi +$$

$$x^2 - 5x + 6 \geq 0 \to (x-3)(x-2)$$

$$\begin{array}{ccccc} & 2 & & 3 & \\ + & | & - & | & + \end{array} \quad D_f = (-\infty, 2) \cup (3, +\infty)$$

$$x^2 - 6x + 5 < 0 \to (x-5)(x-1) \quad \begin{array}{ccccc} & 1 & & 5 & \\ + & | & - & | & + \end{array}$$

$$D_f = (1, 5)$$

$$x^2 + 6x + 5 \geq 0 \to (x+1)(x+5) \quad \begin{array}{ccccc} & -5 & & -1 & \\ + & | & - & | & + \end{array}$$

$$D_f = (-\infty, 5] \cup [-1, +\infty)$$

$$x^2 - 7x + 6 \leq 0 \to (x-4)(x-3) \quad \begin{array}{cc} 3 & 4 \\ | & | \end{array}$$

$$D_f = [3, 4]$$

$$y = \frac{x^2 - 3x + 2}{x - 5} \geq 0 \quad (x-1)(x-2) \quad x < 5 \quad \begin{array}{cccccccc} & 1 & & 2 & & 5 & & -5 \\ + & \phi & - & \phi & + & \phi & - & \end{array}$$

$$D_f = (-\infty, 1) \cup (2, 5)$$

MRNOTE

$$y=\frac{x^2-1}{x^2-6x+5}\geq 0 \longrightarrow \frac{x^2\geq 1 \quad x\geq 1,\; x\leq -1}{(x-5)(x-1)}$$

$-1 \quad 1 \quad 5$

$D_f \; (-\infty \quad -1]\cup[1,5]\cup[5,+\infty)$

$$y=\sqrt{\frac{x^2-4x+3}{x^3-1}}\geq 0 \qquad (x-1)(x-3) \qquad -6$$

$x(x^2-1)\longrightarrow x(x+1)(x-1)$

$-1 \quad 0 \quad 1 \quad 3$

$- \; + \; - \; - \; +$

$D_f=(-1,0)\cup[3,+\infty)$

$$y=\sqrt[3]{\frac{x^2-4x+3}{x^2-1}}\geq 0 \qquad \frac{(x-1)(x-3)}{(x-1)(x+1)}$$

$-1 \quad 1 \quad 3$

$+ \; - \; - \; +$

$D_f=(-\infty,-1)\cup[3,+\infty)$

$$y=\log^{(x^2-3x)} \qquad x^2-3x>0 \qquad -7$$

$x(x-3)>0$

$0 \quad 3$

$+ \; - \; +$

$D_f=(-\infty,0)\cup(3,+\infty)$

$$y=\log_{|x|-2}^{16-x^2}$$

$(x-4)(x+4)<0 \longrightarrow -4<x<4$

$|x|-2\neq 1 \longrightarrow |x|\neq 3$

$|x|-2>0 \longrightarrow |x|>2$

$-4 \quad 2 \quad 3 \quad 4$

$- \; + \; - \; + \; -$

$D_f=(-4,2)\cup(3,4)$

$$y=\log_{10-x}\frac{x^2-7x+12}{x-3}$$

$(x-3)(x-4)$ $\quad 3 \quad 4$

$x>3$

$10-x>0 \longrightarrow 10>x$

$10-x\neq 1 \longrightarrow 9\neq x$

$3 \quad 4 \quad 9 \quad 10$

$- \; - \; + \; - \; +$

$D_f=(4,9)\cup(10,+\infty)$

$$y=\sqrt{x^2-11x+28}+\log_x\sqrt{36-x^2}$$

$\sqrt{x^2-11x+28}\geq 0 \quad (x-4)(x-7) \quad 4 \quad 7$

$\log_x\sqrt{36-x^2}>0 \quad (x+6)(x-6) \quad -6 \quad +6$

$-6 \quad 4 \quad 6 \quad 7$

$D_f=$

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$$y=\frac{x^2-x}{x^2+x}=\frac{x(x-1)}{x(x+1)}$$

$x \to 0$, $x \to -1$

		-1		0		1	
	$+$	$\nexists$	$-$	$\nexists$	$-$	$\mid$	$+$

		-1		0		1	
x^2-x	$+$		$+$	0	$-$	0	$+$
x^2+x	$+$	$\nexists$	$-$	$\nexists$	$+$		$+$
$P(x)$	$+$		$-$		$-$		$+$

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$$y=\sqrt{\frac{x-f(x)}{f(x)}}$$

$(f(x)>0,\ x\geq f(x))\,(f(x)<0,\ x\leq f(x))$

$f(x)>0$

$\frac{x-f(x)}{f(x)}\geq 0 \Rightarrow x-f(x)\geq 0 \to x\geq f(x)$

$f(x)<0$

$\frac{x-f(x)}{f(x)}\geq 0 \Rightarrow x-f(x)\leq 0 \to x\leq f(x)$

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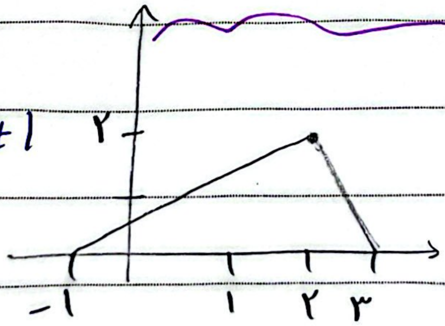
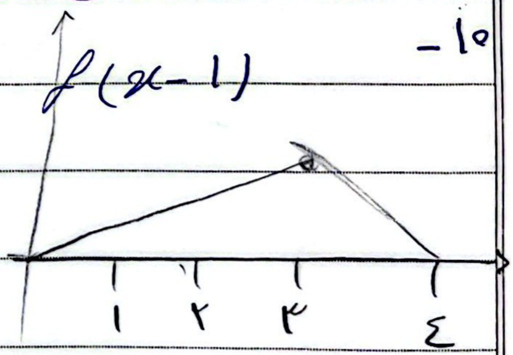
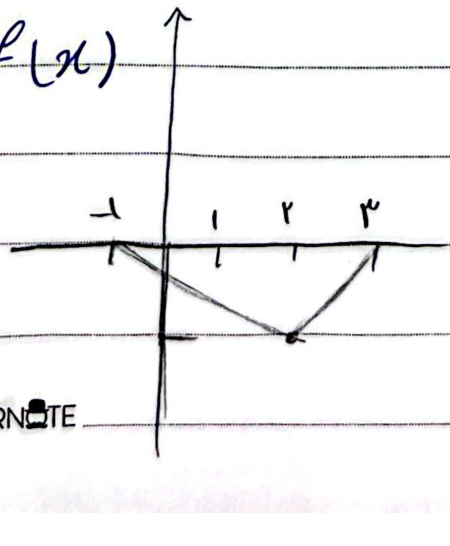
$f(x+1)$



$f(x-1)$



$-f(x)$



$f(x+1)+2$