

الف) $\frac{x+2}{(x-1)(x^2-14x+15)}$ $\rightarrow P_f = IR - \{1, \frac{5}{2}, \frac{3}{2}\}$ $\frac{x^3-20x^2+31x-15}{x^2-14x+15}$

$\frac{x^3-20x^2+31x-15}{x^2-14x+15} = \frac{16}{x} + \frac{14x-15}{x^2-14x+15}$

$\frac{14x-15}{x^2-14x+15} = \frac{14}{x} + \frac{-15}{x^2-14x+15}$

$\frac{-15}{x^2-14x+15} = \frac{-15}{(x-1)(x-15)}$

$\frac{-15}{(x-1)(x-15)} = \frac{A}{x-1} + \frac{B}{x-15}$

$\frac{-15}{(x-1)(x-15)} = \frac{A(x-15)}{(x-1)(x-15)} + \frac{B(x-1)}{(x-1)(x-15)}$

$\frac{-15}{(x-1)(x-15)} = \frac{A(x-15) + B(x-1)}{(x-1)(x-15)}$

$-15 = A(x-15) + B(x-1)$

$-15 = Ax - 15A + Bx - B$

$-15 = (A+B)x - 15A - B$

$0 = A+B$ (1)

$-15 = -15A - B$ (2)

(1) $\rightarrow B = -A$

(2) $\rightarrow -15 = -15A - (-A) = -15A + A = -14A$

$A = \frac{15}{14}$

$B = -\frac{15}{14}$

$\frac{-15}{(x-1)(x-15)} = \frac{\frac{15}{14}}{x-1} - \frac{\frac{15}{14}}{x-15}$

$\frac{14x-15}{x^2-14x+15} = \frac{14}{x} + \frac{\frac{15}{14}}{x-1} - \frac{\frac{15}{14}}{x-15}$

$\frac{x^3-20x^2+31x-15}{x^2-14x+15} = \frac{16}{x} + \frac{14x-15}{x^2-14x+15}$

$\frac{x^3-20x^2+31x-15}{x^2-14x+15} = \frac{16}{x} + \frac{14}{x} + \frac{\frac{15}{14}}{x-1} - \frac{\frac{15}{14}}{x-15}$

$\frac{x^3-20x^2+31x-15}{x^2-14x+15} = \frac{30}{x} + \frac{\frac{15}{14}}{x-1} - \frac{\frac{15}{14}}{x-15}$

$\frac{x^3-20x^2+31x-15}{x^2-14x+15} = \frac{30}{x} + \frac{\frac{15}{14}}{x-1} - \frac{\frac{15}{14}}{x-15}$

الف) $\frac{x+1}{x-\sqrt{3-2x}}$ $\rightarrow \begin{cases} 3-2x \geq 0 \rightarrow 2x \leq 3 \rightarrow x \leq \frac{3}{2} \text{ (I)} \\ x-\sqrt{3-2x} \neq 0 \rightarrow \sqrt{3-2x} \neq x \rightarrow x^2 \neq 3-2x \rightarrow x^2+2x-3 \neq 0 \rightarrow (x-1)(x+3) \neq 0 \end{cases}$

ب) $P_f = (-\infty, 1.5] - \{1\}$ $\leftarrow I \cap II$

ج) $\begin{cases} 3-2x \geq 0 \rightarrow 2x \leq 3 \rightarrow x \leq \frac{3}{2} \text{ (I)} \\ x-\sqrt{3-2x} \neq 0 \rightarrow x \neq \sqrt{3-2x} \rightarrow x^2 \neq 3-2x \rightarrow x^2+2x-3 \neq 0 \rightarrow (x-1)(x+3) \neq 0 \end{cases}$

$I \cap II \rightarrow P_f = [\frac{3}{2}, +\infty) - \{1, 2\}$

الف) $\sqrt{\cos x - 1} \neq 0 \rightarrow \sqrt{\cos x} \neq 1 \rightarrow \cos x \neq 1$ $\odot P_f = IR - \{2k\pi + \frac{\pi}{2}, 2k\pi - \frac{\pi}{2}\}$

ب) $\sqrt{\sin x + 1} \neq 0 \rightarrow \sqrt{\sin x} \neq -1 \rightarrow \sin x \neq -1$ $\odot P_f = IR - \{2k\pi - \frac{\pi}{2}\}$

ج) $\cot x - 1 \neq 0 \rightarrow \cot x \neq 1 \rightarrow \frac{\cos x}{\sin x} \neq 1 \rightarrow \cos x \neq \sin x$ $\odot P_f = IR - \{k\pi + \frac{\pi}{4}, k\pi - \frac{\pi}{4}\}$

$\sin x \neq 0 \rightarrow \odot P_f = IR - \{k\pi\}$

الف) $x^2-5x+4 > 0 \rightarrow (x-2)(x-4) > 0 \rightarrow \frac{0}{+} - \frac{0}{+} + \frac{0}{+} \rightarrow P_f = (2, 4)$

ب) $(x-1)(x-5) < 0 \rightarrow \frac{0}{+} - \frac{0}{+} + \frac{0}{+} \rightarrow P_f = (1, 5)$

ج) $x^2+4x+5 \geq 0 \rightarrow (x+1)(x+5) \geq 0 \rightarrow \frac{0}{+} - \frac{0}{-} - \frac{0}{+} \rightarrow P_f = [-1, +\infty) \cup (-\infty, -5]$

د) $(x-1)(x-4) < 0 \rightarrow \frac{0}{+} - \frac{0}{+} + \frac{0}{+} \rightarrow P_f = [1, 4]$

الف) $\frac{(x-1)(x-2)}{x-5} < 0 \rightarrow \frac{0}{-} - \frac{0}{+} - \frac{0}{+} \rightarrow P_f = (-\infty, 1) \cup (2, 5)$

ب) $\frac{(x-1)(x+1)}{(x-1)(x-5)} \geq 0 \rightarrow \frac{0}{+} - \frac{0}{-} - \frac{0}{+} \rightarrow P_f = (-\infty, -1] \cup (5, +\infty)$

الف) $\frac{(x-1)(x-3)}{x^2-1} \geq 0 \rightarrow \frac{1}{-1} - \frac{3}{1} \rightarrow D_f = [1, +\infty)$

ب) $x^2-1 \neq 0 \rightarrow (x-1)(x+1) \neq 0 \rightarrow D_f = \mathbb{R} - \{ \pm 1 \}$

الف) $x^2-3x > 0 \rightarrow x(x-3) > 0 \rightarrow \frac{0}{-3} - \frac{3}{0} \rightarrow D_f = (3, +\infty) \cup (-\infty, 0)$

ب) $14-x^2 > 0 \rightarrow (x-4)(x+4) > 0 \rightarrow \frac{4}{-4} - \frac{-4}{0} \rightarrow (-4, 4)$

$|x|-2 > 0 \rightarrow |x| > 2 \rightarrow x > 2, x < -2 \rightarrow D_f = (-\infty, -2) \cup (2, +\infty)$

ج) $\frac{x^2-4x+12}{x-2} > 0 \rightarrow \frac{(x-2)(x-6)}{x-2} > 0 \rightarrow x \neq \pm 2$

$10-x > 0 \rightarrow x < 10 \rightarrow D_f = (-\infty, 10)$

$10-x \neq 1 \rightarrow x \neq 9 \rightarrow D_f = (-\infty, 10) - \{9\}$

$\frac{x(x-1)}{x^2+x} \rightarrow \frac{x(x-1)}{x(x+1)} \rightarrow \frac{0}{-1} - \frac{1}{0} \rightarrow \frac{0}{-1} - \frac{1}{0}$

الف) $\rightarrow$

	-1	0	1
x^2-x	+	+	+
x^2+x	+	-	+
$\frac{x^2-x}{x^2+x}$	+	-	+

$\frac{x-f(x)}{f'(x)} \geq 0 \rightarrow \frac{0}{-1} - \frac{1}{1} \rightarrow D_f = [-1, 1] \cup [1, 3]$

