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الف) $y = \frac{\sin x - 2}{2 \cos x + 1} \Rightarrow 2 \cos x + 1 \neq 0 \Rightarrow \cos x \neq -\frac{1}{2} \Rightarrow D_f = \mathbb{R} - \left\{ 2k\pi + \frac{2\pi}{3}, 2k\pi + \frac{4\pi}{3} \right\}$

ب) $y = \frac{\sin x + 2}{\cos x - 1} \Rightarrow \cos x - 1 \neq 0 \Rightarrow \cos x \neq 1 \Rightarrow D_f = \mathbb{R} - \{ 2k\pi \}$

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الف) $y = \frac{2 \sin x + 1}{\tan x + 1} \Rightarrow \tan x + 1 \neq 0 \Rightarrow \tan x \neq -1 \Rightarrow D_f = \mathbb{R} - \left\{ k\pi + \frac{\pi}{4}, k\pi + \frac{5\pi}{4} \right\}$

ب) $y = \frac{\cos x + 1}{\cot x - 1} \Rightarrow \cot x - 1 \neq 0 \Rightarrow \cot x \neq 1 \Rightarrow D_f = \mathbb{R} - \left\{ k\pi + \frac{\pi}{4}, k\pi \right\}$

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الف) $\sin y = x^2 - 2 \Rightarrow -1 \leq x^2 - 2 \leq 1 \Rightarrow 1 \leq x^2 \leq 3 \Rightarrow 1 \leq x \leq \sqrt{3} \text{ or } -\sqrt{3} \leq x \leq -1 \Rightarrow D_f = [-\sqrt{3}, -1] \cup [1, \sqrt{3}]$

ب) $y = \arccos(\sqrt{x} - 3) \Rightarrow -1 \leq \sqrt{x} - 3 \leq 1 \Rightarrow 2 \leq \sqrt{x} \leq 4 \Rightarrow 4 \leq x \leq 16 \Rightarrow D_f = [4, 16]$

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الف) $\cos y = |x| - 3 \Rightarrow -1 \leq |x| - 3 \leq 1 \Rightarrow 2 \leq |x| \leq 4 \Rightarrow 2 \leq x \leq 4 \text{ or } -4 \leq x \leq -2 \Rightarrow D_f = [-4, -2] \cup [2, 4]$

ب) $y = \arcsin(x^2 + 3x + 1) \Rightarrow -1 \leq x^2 + 3x + 1 \leq 1 \Rightarrow \begin{cases} x^2 + 3x + 1 \leq 1 \\ x^2 + 3x + 1 \geq -1 \end{cases} \Rightarrow \begin{cases} x^2 + 3x \leq 0 \\ x^2 + 3x + 2 \geq 0 \end{cases} \Rightarrow \begin{cases} x(x+3) \leq 0 \\ (x+1)(x+2) \geq 0 \end{cases} \Rightarrow D_f = [-3, -2] \cup [-1, 0]$

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الف) $y = \log_3 \frac{x^2 - 4}{x} \Rightarrow \frac{x^2 - 4}{x} > 0 \Rightarrow \frac{x^2}{x} > 4 \Rightarrow \frac{x}{1} > 4 \Rightarrow D_f = (-\infty, -4) \cup (4, +\infty)$

ب) $y = \log_2 \frac{2 - |x|}{x} \Rightarrow 2 - |x| > 0 \Rightarrow |x| < 2 \Rightarrow -2 < x < 2 \Rightarrow D_f = (-2, 2)$

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الف) $y = \log_{x-3} \frac{\omega - x}{x-3} \Rightarrow \begin{cases} \omega - x > 0 \\ x-3 > 0 \\ x-3 \neq 1 \end{cases} \Rightarrow \begin{cases} x < \omega \\ x > 3 \\ x \neq 4 \end{cases} \Rightarrow D_f = (3, \omega) - \{4\}$

ب) $y = \log_{x+3} \frac{x^2 - 1}{x+3} \Rightarrow \begin{cases} x^2 - 1 > 0 \\ x+3 > 0 \\ x+3 \neq 1 \end{cases} \Rightarrow \begin{cases} x > 1 \\ x > -3 \\ x \neq -2 \end{cases} \Rightarrow D_f = (-3, -2) \cup (1, +\infty) - \{-2\}$

-V

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الف) $y = \log \frac{x^2 - 4x + 3}{x - 2} = \log \frac{(x-1)(x-3)}{x-2} > 0 \Rightarrow \frac{(x-1)(x-3)}{x-2} > 1 \Rightarrow \frac{x^2 - 4x + 3}{x-2} > 1$

ب) $y = \log \frac{x+3}{x-2} = \log \frac{x+3}{x-2} > 0 \Rightarrow \frac{x+3}{x-2} > 1 \Rightarrow x+3 > x-2 \Rightarrow 5 > 0$
 $x+3 \neq 0 \Rightarrow x \neq -3$
 $x-2 \neq 0 \Rightarrow x \neq 2$
 $\Rightarrow (-\infty, -3) \cup (2, +\infty) - \{-3\}$

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الف) $y = \sqrt{1 - \log_2(x-1)} = \sqrt{1 - \log_2(x-1)} \geq 0 \Rightarrow 1 - \log_2(x-1) \geq 0 \Rightarrow \log_2(x-1) \leq 1 \Rightarrow x-1 \leq 2 \Rightarrow x \leq 3$
 $x-1 > 0 \Rightarrow x > 1$
 $\Rightarrow D_f = (1, 3]$

ب) $y = \log(2 \log_3 x - 1) = \log(2 \log_3 x - 1) \geq 0 \Rightarrow 2 \log_3 x - 1 \geq 1 \Rightarrow 2 \log_3 x \geq 2 \Rightarrow \log_3 x \geq 1 \Rightarrow x \geq 3$
 $2 \log_3 x - 1 > 0 \Rightarrow \log_3 x > \frac{1}{2} \Rightarrow x > \sqrt{3}$
 $\Rightarrow D_f = (\sqrt{3}, +\infty)$

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الف) $y = \frac{x}{x^2 + 1} = \frac{x}{x^2 + 1} \neq 0 \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$

ب) $y = \frac{x}{x^2 - 1} = \frac{x}{x^2 - 1} \neq 0 \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$

ج) $y = \frac{x}{x^2 - 2} = \frac{x}{x^2 - 2} \neq 0 \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$

د) $y = \frac{x}{x^2 - 3} = \frac{x}{x^2 - 3} \neq 0 \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$

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الف) $y = (x+1)! = (x+1)! \in \mathbb{W} \Rightarrow x+1 \in \mathbb{W} \Rightarrow x \in \mathbb{W} - 1 \Rightarrow D_f = \{x \mid x = k-1, k \in \mathbb{W}\}$

ب) $y = \left(\frac{x^2 - 2}{x - 2}\right)! = \frac{x^2 - 2}{x - 2} \in \mathbb{W} \Rightarrow \frac{x^2 - 2}{x - 2} \in \mathbb{W} \Rightarrow x^2 - 2 = (x-2)k \Rightarrow x^2 - 2 = x^2 - 2x + 2k \Rightarrow -2 = -2x + 2k \Rightarrow x = k - 1$
 $\Rightarrow D_f = \{x \mid x = k-1, k \in \mathbb{W}\}$