

①

$$\text{الف) } y = \frac{\sin x - 2}{2 \cos x + 1} \rightarrow 2 \cos x \neq -1 \rightarrow \cos x \neq -\frac{1}{2} \quad D_f = \mathbb{R} - \left\{ 2k\pi + \frac{2\pi}{3}, 2k\pi + \frac{4\pi}{3} \right\}$$

$$\text{ب) } y = \frac{\sin x + 1}{\cos x - 1} \rightarrow \cos x \neq 1 \rightarrow D_f = \mathbb{R} - \{ 2k\pi \}$$

$$\text{الف) } y = \frac{2 \sin x + 1}{\tan x + 1} \rightarrow \tan x \neq -1 \rightarrow k\pi - \frac{\pi}{4} \quad D_f = \left\{ k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4} \right\} \quad \text{②}$$

$$\tan x = \frac{\sin x}{\cos x} \rightarrow \cos x \neq 0 \rightarrow k\pi + \frac{\pi}{2}$$

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$$\text{ب) } y = \frac{\cos x + 1}{\cot x - 1} \rightarrow \cot x \neq 1 \rightarrow k\pi + \frac{\pi}{4} \quad D_f = \mathbb{R} - \left\{ k\pi - \frac{\pi}{4}, k\pi \right\}$$

$$\cot x = \frac{\cos x}{\sin x} \rightarrow \sin x \neq 0 \rightarrow k\pi$$

$$\cot x = \frac{\cos x}{\sin x} \rightarrow \sin x \neq 0$$

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$$\text{الف) } \sin y = x^2 - 2 \rightarrow -1 \leq x^2 - 2 \leq 1 \rightarrow 1 \leq x^2 \leq 3 \rightarrow 1 \leq x \leq \sqrt{3} \Rightarrow D_f = [1, \sqrt{3}] \quad \text{③}$$

$$\text{ب) } y = \arccos(\sqrt{x} - 3) \rightarrow \sqrt{x} - 3 \geq 0 \quad (I) \rightarrow -1 \leq \sqrt{x} - 3 \leq 1 \rightarrow 2 \leq \sqrt{x} \leq 4 \rightarrow 4 \leq x \leq 16 \quad (II)$$

$$D_f = I \cap II \Rightarrow D_f = [4, 16]$$

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$$\text{الف) } \cos y = |x| - 3 \rightarrow -1 \leq |x| - 3 \leq 1 \Rightarrow 2 \leq |x| \leq 4 \rightarrow 2 \leq x \leq 4 \text{ or } -4 \leq x \leq -2$$

$$D_f = [-4, -2] \cup [2, 4]$$

$$\text{ب) } y = \arcsin(x^2 + 3x + 1) \rightarrow -1 \leq x^2 + 3x + 1 \leq 1 \Rightarrow \begin{cases} 0 \leq x^2 + 3x + 2 \\ 0 \leq (x+1)(x+2) \\ x^2 + 3x \leq 0 \\ x(x+3) \leq 0 \end{cases}$$

$$\begin{matrix} -2 & -1 & 0 \\ + & - & + \end{matrix} \quad \begin{matrix} -3 & 0 & + \\ + & - & + \end{matrix} \quad \begin{matrix} -2 & -1 & 0 \\ + & - & + \end{matrix} \quad \begin{matrix} -3 & 0 & + \\ + & - & + \end{matrix}$$

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$$D_f = I \cap II \Rightarrow D_f = [-3, -2] \cup [-1, 0]$$

$$\text{الف) } y = \log_r x^2 - 4 \rightarrow x^2 - 4 > 0 \rightarrow x^2 > 4 \rightarrow x > 2 \text{ or } x < -2 \Rightarrow D_f = (-\infty, -2) \cup (2, +\infty) \quad \text{④}$$

$$\text{ب) } y = \log_r |x| \rightarrow 2 - |x| > 0 \rightarrow |x| < 2 \Rightarrow -2 < x < 2 \Rightarrow D_f = (-2, 2)$$

الف) $y = \log_{x-3}^{x-5}$ $\begin{cases} x-5 > 0 \rightarrow x > 5 \Rightarrow x \in (5, \infty) \text{ (I)} \\ x-3 > 0 \rightarrow x > 3 \text{ (II)} \\ x-3 \neq 1 \rightarrow x \neq 4 \text{ (III)} \end{cases} \Rightarrow D_f = (3, 5) - \{4\}$ ④

ب) $y = \log_{x+3}^{x^2-1}$ $\begin{cases} x^2-1 > 0 \Rightarrow x^2 > 1 \Rightarrow \begin{cases} x > 1 \\ x < -1 \end{cases} \rightarrow (-\infty, -1) \cup (1, +\infty) \text{ (I)} \\ x+3 > 0 \rightarrow x > -3 \\ x+3 \neq 1 \Rightarrow x \neq -2 \rightarrow (-3, \infty) - \{-2\} \text{ (II)} \end{cases} \Rightarrow D_f = (I) \cap (II) \Rightarrow D_f = (-3, -1) - \{-2\} \cup (1, +\infty)$ 5

الف) $y = \log_{x-3}^{(x-3)(x-1)}$ $\rightarrow \frac{(x-3)(x-1)}{(x-3)} > 0 \rightarrow x-1 > 0 \rightarrow x > 1$ ⑤

$D_f = (1, 3) \cup (3, +\infty) \Rightarrow D_f = (1, +\infty) - \{3\}$ 10

ب) $y = \log_{x+5}^{\frac{x+3}{x-2}}$ $\begin{cases} \frac{x+3}{x-2} > 0 \rightarrow \begin{cases} x > 2 \\ x < -3 \end{cases} \rightarrow (-\infty, -3) \cup (2, +\infty) \text{ (I)} \\ x+5 > 0 \rightarrow x > -5 \text{ (II)} \\ x+5 \neq 1 \Rightarrow x \neq -4 \text{ (III)} \end{cases} \Rightarrow D_f = (I) \cap (II) \cap (III) = \begin{cases} D_f = (-5, -3) - \{-4\} \\ \cup (2, +\infty) \end{cases}$ 15

الف) $y = \sqrt{3 - \log_2(x-2)}$ $\rightarrow 3 - \log_2(x-2) \geq 0 \rightarrow \log_2(x-2) \leq 3 \rightarrow \begin{cases} x-2 \leq 8 \\ x-2 \geq 1 \end{cases} \Rightarrow D_f = (2, 10]$ ①

ب) $y = \log_2(2 \log_2^x - 1)$ $\begin{cases} 2 \log_2^x - 1 > 0 \rightarrow 2 \log_2^x > 1 \rightarrow \log_2^x > \frac{1}{2} \rightarrow x > \sqrt{2} \end{cases} \Rightarrow D_f = (\sqrt{2}, +\infty)$ 20

الف) $\frac{x}{x^2+1}$ $\rightarrow \frac{x}{x^2+1} \neq 0 \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$ ②

ب) $\frac{x}{x^2+1}$ $\rightarrow \frac{x}{x^2+1} \neq 1 \Rightarrow x \neq x^2+1 \Rightarrow x^2-x+1 \neq 0 \Rightarrow D_f = \mathbb{R} - \{\frac{1 \pm \sqrt{5}}{2}\}$ 25

الف) $y = (kx+1)^{\frac{1}{k}}$ $\rightarrow kx+1 \in \mathbb{N} \rightarrow kx \in \mathbb{N}-1 \rightarrow x \in \frac{\mathbb{N}-1}{k}$ ③

ب) $y = \left(\frac{kx-1}{x-2}\right)!$ $\rightarrow \frac{kx-1}{x-2} \in \mathbb{N} \Rightarrow kx-1 = (x-2)k \Rightarrow kx - kx + 2k = 1 \Rightarrow 2k = 1 \Rightarrow k = \frac{1}{2}$ 30