

$$y = \frac{\sin x - 1}{1 + \cos x}$$

$$D_f = \mathbb{R} - \left\{ 2k\pi + \frac{3\pi}{2}, 2k\pi + \frac{5\pi}{2} \right\} \quad (\text{الف})$$

$$1 + \cos x \neq 0 \Rightarrow \cos x \neq -1 \Rightarrow x \neq \pi, 3\pi, \dots$$

$$y = \frac{\sin x + 1}{\cos x - 1}$$

$$\Rightarrow \cos x - 1 \neq 0 \Rightarrow \cos x \neq 1 \Rightarrow x \neq 0, 2\pi, \dots \quad D_f = \mathbb{R} - \{2k\pi\} \quad (\text{ب})$$

$$y = \frac{1 + \sin x}{\tan x + 1}$$

$$\uparrow \text{مخرج} = \frac{\sin x}{\cos x} + 1 \Rightarrow \cos x \neq 0 \Rightarrow x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

$$\tan x \neq -1 \Rightarrow x \neq \frac{3\pi}{4}, \frac{7\pi}{4}, \dots$$

$$D_f = \mathbb{R} - \left\{ k\pi + \frac{\pi}{2}, k\pi + \frac{3\pi}{4} \right\} \quad (\text{الف})$$

$$y = \frac{\cos x + 1}{\cot x - 1}$$

$$\textcircled{1} \sin x \neq 0 \Rightarrow x \neq 0, \pi, 2\pi, \dots$$

$$\Rightarrow \frac{\cos x}{\sin x} - 1 \neq 0 \Rightarrow \cot x - 1 \neq 0 \Rightarrow \cot x \neq 1 \Rightarrow x \neq \frac{\pi}{4}, \frac{5\pi}{4}, \dots \quad D_f = \mathbb{R} - \left\{ k\pi, k\pi + \frac{\pi}{4} \right\} \quad (\text{ب})$$

$$\sin y = x^2 - 2$$

$$-1 \leq \sin y \leq 1$$

$$-1 \leq x^2 - 2 \leq 1$$

$$1 \leq x^2 \leq 3 \Rightarrow x \leq -\sqrt{3} \text{ یا } x \geq \sqrt{3}$$

$$D_f = [-\sqrt{3}, -1] \cup [1, \sqrt{3}] \quad (\text{الف})$$

$$y = \arccos(\sqrt{x} - 3)$$

$$-1 \leq \sqrt{x} - 3 \leq 1 \Rightarrow 2 \leq \sqrt{x} \leq 4 \Rightarrow 4 \leq x \leq 16 \quad (\text{ب})$$

$$D_f = [4, 16]$$

$$\cos y = |x| - 3$$

$$-1 \leq |x| - 3 \leq 1$$

$$2 \leq |x| \leq 4 \Rightarrow x \leq -2 \text{ یا } x \geq 2 \quad (\text{الف})$$

$$D_f = [-4, -2] \cup [2, 4]$$

$$y = \arcsin(x^2 + 3x + 1)$$

$$-1 \leq x^2 + 3x + 1 \leq 1$$

$$\left\{ \begin{array}{l} x^2 + 3x + 1 \leq 1 \Rightarrow x(x+3) \leq 0 \\ x^2 + 3x + 1 \geq -1 \Rightarrow (x+1)(x+2) \geq 0 \end{array} \right. \quad (\text{ب})$$

$$\begin{array}{c} -1 \leq 0 \\ + \quad - \quad + \end{array} \Rightarrow [-3, 0]$$

$$\begin{array}{c} -2 \leq -1 \\ + \quad - \quad + \end{array} \Rightarrow (-\infty, -2] \cup [-1, +\infty)$$

$$D_f = [-3, -2] \cup [-1, 0] \quad (\text{الف})$$

$$y = \log_{\frac{1}{2}}(x^2 - 4)$$

$$x^2 - 4 > 0$$

$$x^2 > 4$$

$$\begin{array}{c} x > 2 \\ x < -2 \end{array}$$

$$D_f = (-\infty, -2) \cup (2, +\infty) \quad (\text{الف})$$

$$y = \log_2 \frac{2 - |x|}{2}$$

$$2 - |x| > 0$$

$$|x| < 2$$

$$-2 < x < 2$$

$$D_f = (-2, 2) \quad (\text{ب})$$

$$y = \log \frac{\alpha - x}{x - \beta} \quad \left. \begin{array}{l} \alpha - x > 0 \\ x - \beta > 0 \\ x - \beta \neq 1 \end{array} \right\} \Rightarrow D_f = (\beta, \alpha) - \{1\} \quad (\text{الف})$$

$$y = \log \frac{x^2 - 1}{x + \beta} \quad \left. \begin{array}{l} (x^2 - 1)(x + \beta) > 0 \\ x + \beta > 0 \\ x + \beta \neq 1 \end{array} \right\} \Rightarrow D_f = ((-\beta, -1) \cup (1, +\infty)) - \{-\beta\} \quad (\text{ب})$$

$$y = \log \frac{x^2 - \varepsilon x + \beta}{x - \beta} \quad \frac{(x-1)(x-\beta)}{(x-\beta)} > 0 \Rightarrow D_f = (1, \beta) \cup (\beta, +\infty) \quad (\text{الف})$$

$$y = \log \frac{x + \beta}{x - \beta} \quad \left. \begin{array}{l} \frac{x + \beta}{x - \beta} > 0 \\ x + \beta > 0 \\ x + \beta \neq 1 \end{array} \right\} \Rightarrow D_f = ((-\infty, -\beta) \cup (\beta, +\infty)) - \{-\beta\} \quad (\text{ب})$$

$$y = \sqrt{x} - \log_r x \quad \left. \begin{array}{l} x - \log_r x > 0 \\ x - \beta > 0 \end{array} \right\} \Rightarrow D_f = \{x, 1\} \quad (\text{الف})$$

$$y = \log (x \log_r x - 1) \quad \left. \begin{array}{l} x \log_r x - 1 > 0 \\ x \log_r x > 1 \\ \log_r x > \frac{1}{x} \end{array} \right\} \Rightarrow D_f = (\sqrt{x}, +\infty) \quad (\text{ب})$$

$$y = \frac{x}{\varepsilon x + 1} \quad \varepsilon x + 1 \neq 0 \Rightarrow \varepsilon x \neq -1 \Rightarrow D_f = \mathbb{R} \quad (\text{الف})$$

$$y = \frac{x}{\varepsilon x - 1} \quad \varepsilon x - 1 \neq 0 \quad \varepsilon x \neq 1 \Rightarrow x \neq \frac{1}{\varepsilon} \Rightarrow D_f = \mathbb{R} - \{\frac{1}{\varepsilon}\} \quad (\text{ب})$$

$$y = \frac{x}{\varepsilon x - \beta} \quad \varepsilon x - \beta \neq 0 \quad \varepsilon x \neq \beta \Rightarrow x \neq \frac{\beta}{\varepsilon} \Rightarrow D_f = \mathbb{R} - \{\frac{\beta}{\varepsilon}\} \quad (\text{ج})$$

$$y = \frac{x}{\varepsilon x - \beta} \quad \varepsilon x - \beta \neq 0 \quad \varepsilon x \neq \beta \Rightarrow x \neq \log_{\varepsilon} \beta \Rightarrow D_f = \mathbb{R} - \{\log_{\varepsilon} \beta\} \quad (\text{د})$$

$$y = (\varepsilon x + 1)! \quad \varepsilon x + 1 \in \mathbb{N} \Rightarrow \varepsilon x \in \mathbb{N} - 1 \Rightarrow x \in \frac{\mathbb{N} - 1}{\varepsilon} \quad (\text{الف})$$

$$D_f = \{x \mid x = \frac{k-1}{\varepsilon}, k \in \mathbb{N}\}$$

$$y = \left(\frac{x - \beta}{x - \alpha} \right)! \quad \frac{x - \beta}{x - \alpha} \in \mathbb{N} \quad x = - \frac{-\alpha \mathbb{N} + \beta}{\mathbb{N} - \beta} = \frac{-\beta + \alpha \mathbb{N}}{\mathbb{N} - \beta}$$

$$D_f = \{x \mid x = \frac{\alpha k - \beta}{k - \beta}, k \in \mathbb{N}\}$$