

۲۰. آفرین!

$$y = \frac{\sin x - 1}{1 + \cos x}$$

$$D_f = \mathbb{R} - \left\{ 2k\pi + \frac{3\pi}{2}, 2k\pi + \frac{5\pi}{2} \right\} \quad (\text{الف})$$

$$1 + \cos x \neq 0 \Rightarrow \cos x \neq -1 \Rightarrow x \neq \pi, 3\pi, \dots$$

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$$y = \frac{\sin x + 3}{\cos x - 1}$$

$$\Rightarrow \cos x - 1 \neq 0 \Rightarrow \cos x \neq 1 \Rightarrow x \neq 0, 2\pi, \dots$$

$$D_f = \mathbb{R} - \{ 2k\pi \}$$

(ب)

$$y = \frac{2 \sin x + 1}{6 \sin x + 1}$$

$$\uparrow \text{مخرج} = \frac{\sin x}{\cos x} + 1 \Rightarrow \cos x \neq 0 \Rightarrow x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

$$\tan x \neq -1 \Rightarrow x \neq \frac{3\pi}{4}, \frac{5\pi}{4}, \dots$$

$$D_f = \mathbb{R} - \left\{ k\pi + \frac{\pi}{4}, k\pi + \frac{5\pi}{4} \right\}$$

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$$y = \frac{\cos x + 1}{\cot x - 1}$$

$$\textcircled{1} \sin x \neq 0 \Rightarrow x \neq 0, \pi, \dots$$

$$\textcircled{2} \cot x - 1 \neq 0 \Rightarrow \cot x \neq 1 \Rightarrow x \neq \frac{\pi}{4}, \frac{5\pi}{4}, \dots$$

$$D_f = \mathbb{R} - \left\{ k\pi, k\pi + \frac{\pi}{4} \right\}$$

(ب)

$$\sin y = x^2 - 2$$

$$-1 \leq \sin y \leq 1$$

$$-1 \leq x^2 - 2 \leq 1$$

$$1 \leq x^2 \leq 3$$

$$1 \leq x \leq \sqrt{3}$$

$$- \sqrt{2} \leq x \leq -1$$

$$D_f = [-\sqrt{3}, -1] \cup [1, \sqrt{3}]$$

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$$y = \arccos(\sqrt{x} - 3)$$

$$-1 \leq \sqrt{x} - 3 \leq 1 \Rightarrow 2 \leq \sqrt{x} \leq 4$$

$$\Rightarrow 4 \leq x \leq 16$$

$$D_f = [4, 16]$$

(ب)

$$\cos y = |x| - 3$$

$$-1 \leq |x| - 3 \leq 1$$

$$2 \leq |x| \leq 4$$

$$-4 \leq x \leq -2$$

$$x \leq -2 \quad \text{استثنای ۱} \quad \text{۲} \quad \text{۱} \quad \text{۲}$$

$$D_f = [-4, -2] \cup [2, 4]$$

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$$y = \arcsin(x^2 + 3x + 1)$$

$$-1 \leq x^2 + 3x + 1 \leq 1$$

$$x^2 + 3x + 1 \leq 1 \Rightarrow x(x + 3) \leq 0$$

$$x^2 + 3x + 1 \geq -1 \Rightarrow (x + 1)(x + 2) \geq 0$$

$$D_f = [-3, -2] \cup [-1, 0]$$

$$-1 \leq x^2 + 3x + 1 \leq 1 \Rightarrow [-3, 0]$$

$$-1 \leq x^2 + 3x + 1 \leq 1 \Rightarrow (-\infty, -2] \cup [-1, +\infty)$$

$$y = \log_{\frac{1}{2}}(x^2 - 4)$$

$$x^2 - 4 > 0$$

$$x^2 > 4$$

$$x > 2$$

$$D_f = (-\infty, -2) \cup (2, +\infty)$$

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$$y = \log_{\frac{1}{2}}(2 - |x|)$$

$$2 - |x| > 0$$

$$|x| < 2$$

$$-2 < x < 2$$

$$D_f = (-2, 2)$$

(ب)

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$$y = \log \frac{\alpha - x}{x - \beta} \quad \left. \begin{array}{l} \alpha - x > 0 \\ x - \beta > 0 \\ x - \beta \neq 1 \end{array} \right\} \Rightarrow D_f = (\beta, \alpha) - \{1\} \quad (\text{الف})$$

$$y = \log \frac{x^2 - 1}{x + \beta} \quad \left. \begin{array}{l} (x^2 - 1)(x + \beta) > 0 \\ x + \beta > 0 \\ x + \beta \neq 1 \end{array} \right\} \Rightarrow D_f = ((-\beta - 1) \cup (1, +\infty)) - \{-\beta\} \quad (\text{ب})$$

$$y = \log \frac{x^2 - \varepsilon x + \beta}{x - \beta} \quad \left. \begin{array}{l} (x-1)(x-\beta) > 0 \\ (x-\beta) > 0 \\ (x-\beta) \neq 1 \end{array} \right\} \Rightarrow D_f = (1, \beta) \cup (\beta, +\infty) \quad (\text{الف})$$

$$y = \log \frac{x + \beta}{x - \beta} \quad \left. \begin{array}{l} \frac{x + \beta}{x - \beta} > 0 \\ x + \beta > 0 \\ x + \beta \neq 1 \end{array} \right\} \Rightarrow D_f = ((-\infty, -\beta) \cup (\beta, +\infty)) - \{-\beta\} \quad (\text{ب})$$

$$y = \sqrt{x} - \log_r \frac{x - \beta}{x - \gamma} \quad \left. \begin{array}{l} x - \log_r \frac{x - \beta}{x - \gamma} > 0 \\ x - \beta > 0 \end{array} \right\} \Rightarrow D_f = [\beta, 1] \quad (\text{الف})$$

$$y = \log (x \log_r \frac{x}{\beta} - 1) \quad \left. \begin{array}{l} x \log_r \frac{x}{\beta} - 1 > 0 \\ x \log_r \frac{x}{\beta} > 1 \\ \log_r \frac{x}{\beta} > \frac{1}{x} \end{array} \right\} \Rightarrow D_f = (\sqrt{x}, +\infty) \quad (\text{ب})$$

$$x > 0, x > \sqrt{x} \Rightarrow D_f = (\sqrt{x}, +\infty)$$

$$y = \frac{x}{\varepsilon x + 1} \quad \varepsilon x + 1 \neq 0 \Rightarrow \varepsilon x \neq -1 \Rightarrow D_f = \mathbb{R} \quad (\text{الف})$$

$$y = \frac{x}{\varepsilon x - 1} \quad \varepsilon x - 1 \neq 0 \quad \varepsilon x \neq 1 \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\} \quad (\text{ب})$$

$$y = \frac{x}{\varepsilon x - \beta} \quad \varepsilon x - \beta \neq 0 \quad \varepsilon x \neq \beta \Rightarrow x \neq \frac{\beta}{\varepsilon} \Rightarrow D_f = \mathbb{R} - \{\frac{\beta}{\varepsilon}\} \quad (\text{ج})$$

$$y = \frac{x}{\varepsilon x - \beta} \quad \varepsilon x - \beta \neq 0 \quad \varepsilon x \neq \beta \Rightarrow x \neq \log_r \frac{\beta}{\varepsilon} \Rightarrow D_f = \mathbb{R} - \{\log_r \frac{\beta}{\varepsilon}\} \quad (\text{د})$$

$$y = (\varepsilon x + 1) \mid \quad \varepsilon x + 1 \in \mathbb{W} \Rightarrow \varepsilon x \in \mathbb{W} - 1 \Rightarrow x \in \frac{\mathbb{W} - 1}{\varepsilon} \quad (\text{الف})$$

$$D_f = \{x \mid x = \frac{k-1}{\varepsilon}, k \in \mathbb{W}\} \quad (\text{ب})$$

$$y = \left(\frac{\beta x - \gamma}{\gamma x - \alpha} \right) \mid \quad \frac{\beta x - \gamma}{\gamma x - \alpha} \in \mathbb{W} \quad x = - \frac{-\alpha \mathbb{W} + \gamma}{\gamma \mathbb{W} - \beta} = \frac{-\gamma + \alpha \mathbb{W}}{\gamma \mathbb{W} - \beta}$$

$$D_f = \{x \mid x = \frac{\alpha k - \gamma}{\gamma k - \beta}, k \in \mathbb{W}\} \quad (\text{ج})$$