۶)

$f(x) = \sqrt{4 - \frac{1}{x^2}}$ $\quad 4 - \frac{1}{x^2} \geq 0 \rightarrow 4 \geq \frac{1}{x^2} \rightarrow x^2 \geq \frac{1}{4}$

$x^2 \neq 0 \rightarrow x \neq 0$ $\quad x \geq \frac{1}{2}$ یا $x \leq -\frac{1}{2}$

$D_f = (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, +\infty)$

---

۷)

$f(x) = \sqrt{mx^2 + 2mx + 1} \rightarrow mx^2 + 2mx + 1 \geq 0$

$m \geq 0$

$\Delta \leq 0 \rightarrow (2m)^2 - 4(m)(1) \leq 0$

$4m(m-1) \leq 0$

	0		1	
+	○	−	○	+

$m = 0$, $m = 1 \rightarrow [0, 1]$

---

۸)

$f(x) = \begin{cases} \frac{4x^2 - 1}{2x - 1} & \rightarrow x \neq a \\ 4x + k & \rightarrow x = \frac{1}{2} \end{cases}$

$g(x) = 2x + 1$

$4x + k = 2x + 1 \qquad a + k = \frac{1}{2}$

$2 + k = 2$

$k = 0$

$2x - 1 = 2a - 1 = 0 \quad a = \frac{1}{2}$

---

۹)

$f(x) \begin{cases} \frac{9x^2 - 4}{3x + 2} & x \neq -\frac{2}{3} \\ 3ax + 2 & x = -\frac{2}{3} \end{cases}$

$\frac{9x^2 - 4}{3x + 2} \Rightarrow 3x + b \rightarrow (3x + b)(3x + 2) = 9x^2 - 4$

$(3x + b)(3x + 2) = (3x - 2)(3x + 2) \quad b = -2$

$g(x) = 3x + b \rightarrow x = -\frac{2}{3} \rightarrow 3ax + 2 = 3x + b$ $\qquad 9 - b = 4 - (-2) = 6$

$3(-\frac{2}{3})a + 2 = 3(-\frac{2}{3}) + b \rightarrow -2a = -6$

$a = 3$

---

۱۰)

$f(x) = \begin{cases} \frac{x^3 - 8}{x - 2} & x \neq 2 \\ 2x^2 + ax & x = 2 \end{cases}$ $\qquad g(x) = x + 2$

$2 \cdot 2^2 + ax = x + 2 \longrightarrow 2a^2 + 2a = 4$

$\rightarrow 2a^2 + 2a - 4 = 0 \quad \Delta = b^2 - 4ac \rightarrow 4 - 4(2)(-4) = 36$

$a = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow \frac{-2 \pm \sqrt{36}}{4}$ $\begin{cases} -2 \\ 1 \end{cases}$

۱)

$f(x)\begin{cases} x^2+2x & x\geq a \rightarrow a^2+2(a) \\ ax-4 & x<a \rightarrow a^2-4 \end{cases}$

$a^2+2a=a^2-4$

$2a=-4$

$a=-2$

---

۲)

$L(x)=2x+b \qquad f(x)=\dfrac{x^2+a}{2x-b}$

$L(3)=3=2(2)+b \qquad f(2)=3=\dfrac{4+a}{4-b} \Rightarrow 15=4+a$

$b=-1 \qquad a=11$

$f(-1)=5$

$f(x)=\dfrac{x^2+11}{2x+1} \Rightarrow f(1)=\dfrac{1+11}{2+1}=\dfrac{12}{3}=4$

---

۳)

$f(a)=\dfrac{4x+1}{2x^2+ax+b} \rightarrow 2x^2+ax+b\neq 0$

$x=-1 \rightarrow 2(-1)^2+a(-1)+b=0 \rightarrow 2-a+b=0$

$x=4 \rightarrow 2(4)^2+a(4)+b=0 \rightarrow 32+4a+b=0$

$\rightarrow \begin{cases} -2+a-b=0 \\ 32+4a+b=0 \end{cases}$

$30=-5a$

$a=-6$

$b=-8$

$f(x)=\dfrac{4x+1}{2x^2-6x-8} \rightarrow f(1) \Rightarrow \dfrac{4(1)+1}{2(1)^2-6(1)-8}=\dfrac{-5}{12}$

---

۴)

$f(x)=\dfrac{x^3-\sqrt{3}}{-4x^2+ax+b} \qquad D_f=R-\{-1\} \rightarrow -(2x+k)^2 = x=-1 \Rightarrow -(2(-1)+k)^2=0$

$k=2$

$\rightarrow -4x^2+ax+b \rightarrow -(4x^2+4xk+k^2)=-4x^2-4xk-k^2=-4x^2+ax+b$

$k=2 \rightarrow -4x^2-8x-4 \Rightarrow -4x^2+ax+b$

$b=-4 \rightarrow a=-8 \qquad b+a=-12$

$x=-1 \rightarrow -4x^2+ax+b=0 \rightarrow -4(-1)^2+b=0 \rightarrow -4-a+b=0 \quad b-a-4$

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۵)

$f(x)=\dfrac{2x}{(x-1)(x^2+mx+1)} \qquad D_f=R-\{1\}$

$x=1$

$x=1 \rightarrow (x^2+mx+1)=0 \rightarrow 1+m+1=0 \Rightarrow m=-2$

$\Delta=b^2-4ac \rightarrow \Delta=m^2-4(1)(1)=m^2-4<0$

$m^2<4 \qquad -2<m<2$