۱) دیانا دلش خواه

الف) $(9, x+2y)$, $(3x-y, -4)$ $\quad \frac{x}{y} = \frac{2}{-3}$

$$\left.\begin{array}{l} 3x-y=9 \\ x-3\ \ x+2y=-4 \end{array}\right\} \Rightarrow \begin{array}{l} 3x-y=9 \\ -3x-6y=12 \end{array} \Rightarrow -7y=21 \Rightarrow y=-3 \Rightarrow x=2$$

$$\left.\begin{array}{l} \frac{1}{x}\cdot\frac{1}{y}=-1 \xrightarrow{\times -xy} y-x=xy \\ \frac{5}{x}-\frac{7}{y}=-3 \xrightarrow{\times -xy} 7x-5y=3xy \end{array}\right.$$

ب) $(-1, -3)$, $(\frac{1}{x}\cdot\frac{1}{y}, \frac{5}{x}-\frac{7}{y})$

$$\Rightarrow \left\{\begin{array}{l} 3y-3x=3xy \\ 7x-5y=3y-3x \end{array}\right. \Rightarrow 10x=15y \rightarrow \frac{x}{y}=\frac{15}{10}=\frac{3}{2}$$

۲)

$f=\{(a, 2a), (1, a+1), (1, -2), (2, b)\}$ $\quad a+1=-2 \rightarrow a=-3$

$f=\{(-3, -6), (1, -2), (2, b)\}$

$$\underbrace{f(a)}_{-6} + 2\underbrace{f(2)}_{2b} = 3\underbrace{f(1)}_{3\times -2} \Rightarrow -6+2b=-6 \Rightarrow 2b=0 \Rightarrow b=0$$

۳)

$f=\{(-1, m^2-3m), (3, 5), (-1, -2), (m+1, 4), (2, 4), (m^2+2, 4m+1)\}$

$m^2-3m=-2 \rightarrow m^2-3m+2=0 \Rightarrow (m-1)(m-2)=0 \Rightarrow m=1 , m=2$

(1 , 2)

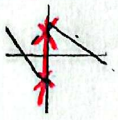
① $m=1 \Rightarrow (m+1, 4) \rightarrow (2, 4) / (m^2+2, 4m+1) \rightarrow (3, 5)$ ✓

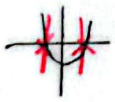
② $m=2 \Rightarrow (m+1, 4) \Rightarrow (3, 4)$ غ ق ق

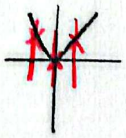
$m=1$

۴)

الف)  تابع نیست چون خط عمود را در نقطه قطع می کند

ج)  تابع نیست چون خط عمود را در ۲ نقطه قطع می کند

ب)  تابع هست

د)  تابع هست

۵)

الف) $y=-\sqrt{x+1} \Rightarrow \left\{\begin{array}{l} y_1=-\sqrt{x+1} \\ y_2=-\sqrt{x+1} \end{array}\right. \Rightarrow y_1=y_2$ تابع هست

ب) $x=\frac{y}{\sqrt{1-y^2}} \xrightarrow{x^2} 2=\frac{y^2}{1-y^2} \Rightarrow 4=\frac{y^2}{1-y^2} \Rightarrow 4-4y^2=y^2 \Rightarrow 4=5y^2 \rightarrow \frac{4}{5}=y^2$

$y=\pm\frac{2}{\sqrt{5}}$

تابع نیست

۶) الف) $|y| = x \xrightarrow{x=1} |y| = 1 \rightarrow y = \pm 1$ تابع نیست

ب) $y^3 + 3y^2 + 3y + x^3 + x = 0 \rightarrow (y+1)^3 - 1 + x^3 + x = 0 \rightarrow (y+1)^3 = 1 - x^3 - x \rightarrow$

$y = \sqrt[3]{1 - x^3 - x} - 1$ $\begin{cases} y_1 = \sqrt[3]{1-x^3-x} - 1 \\ y_2 = \sqrt[3]{1-x^3-x} - 1 \end{cases} \Rightarrow y_1 = y_2$ تابع هست

۷) $f(x) = \dfrac{x^2 + 4x + 5}{x^2 + 2x + 7} \rightarrow \dfrac{(\sqrt{3}-2)^2 + 4(\sqrt{3}-2) + 5}{(\sqrt{3}-2)^2 + 2(\sqrt{3}-2) + 7} = \dfrac{9 - 4\sqrt{3} + 4 + 4\sqrt{3} - 8 + 5}{9 - 4\sqrt{3} + 4 + 2\sqrt{3} - 4 + 7} =$

$\dfrac{10}{12} = \boxed{\dfrac{5}{6}}$

۸) $f(-1) = -4 \rightarrow (-1)^3 + a(-1) + b = -4 \rightarrow b = -3 + a$

$f(x) = 2x - a \rightarrow x^3 + ax + b = 2x - a \xrightarrow{b = -3+a} x^3 + ax + (-3+a) = 2x - a \Rightarrow x^3 + ax - 2x - 3 + a + a = 0$

$\rightarrow x^3 + (a-2)x + (2a-3) = 0 \xrightarrow{x=-1}$ بشاول

$(-1)^3 + (a-2)(-1) + (2a-3) = 0 \Rightarrow (-1+2-3) + (-a+2a) = 0 \Rightarrow -1 + a = 0 \rightarrow \boxed{a=1}$

$b = -3 + a \Rightarrow \boxed{b=-2}$ $x^3 + (a-2)x + (2a-2) = 0 \xrightarrow[b=-2]{a=1} x^3 - 2x - 1 = 0 \xrightarrow{\div x+1} (x+1)(x^2 - x - 1) = 0$

$\Rightarrow x = \dfrac{-(-1) \pm \sqrt{(-1)^2 + 4(1)(1)}}{2(1)} \rightarrow x_2 = \dfrac{1+\sqrt{5}}{2}$, $x_3 = \dfrac{1-\sqrt{5}}{2} \Rightarrow x_2 + x_3 = \dfrac{1+\sqrt{5}+1-\sqrt{5}}{2} = \dfrac{2}{2} = \boxed{1}$

۹) $f_c\{(2a+b), (1, 2a), (-1, a-2b+1)\}$ تابع ثابت است $\Rightarrow \begin{cases} a+b = 2a \Rightarrow b = a \\ a - 2b + 1 = 2a \Rightarrow \end{cases}$

$a - 2a + 1 - 2a = 0 \Rightarrow$

$-3a = -1 \Rightarrow a = \dfrac{-1}{-3} = \boxed{\dfrac{1}{3}}$

۱۰) $\dfrac{4x^2 - ax + c + 1}{bx + 3} = x + \dfrac{bx + 3}{} \rightarrow 4x^2 - ax + c + 1 = bx^2 + 3x$ ضرایب تمام توان‌ها باید برابر صفر باشد

① $\underbrace{4 - b = 0}_{\text{ضریب } x^2} \rightarrow \boxed{b = 4}$ ② $\underbrace{-a - 3 = 0}_{\text{ضریب } x} \boxed{a = -3}$ ③ $\underbrace{c + 1 = 0}_{\text{عدد ثابت}} \boxed{c = -1}$

$a + b + c = -3 + 4 - 1 = 0$