۱) دیانا وطن خواه

۱۹/۵ آفرین

$x^2 - ax + b$ $\quad 1 - a + b = 0$

$1 < x < 3$ $\quad 9 - 3a + b = 0$

$b - a = -1$

$a + b = 7$ ۲

$b - 3a = -9$

x		1		3	
	$+$	0	$-$	0	$+$

$b = a - 1$

$b = 3a - 9$

$a - 1 = 3a - 9 \Rightarrow 2a = 8 \quad a = 4$

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۲)

x		-1		2	
P	$+$	0	$+$	0	$-$

$(-k + 2 + m - 1)(-1 - 3n)^2 = 0$ $\qquad (4k - 8 + m - 1)(4 - 3n)^2 = 0$

$(m - k + 1)(-1 - 3n)^2 = 0$ $\qquad (4k - 9 + m)(4 - 3n)^2 = 0 \Rightarrow$ درست نیست

یا $\begin{cases} m - k = -1 \rightarrow m = k - 1 \ ✗ \\ n = -1/3 \ ✗ \end{cases}$ $\qquad$ یا $\begin{cases} 4k + m = 9 \rightarrow m = 9 - 4k \ ✗ \\ n = 4/3 \ ✗ \end{cases}$

$y = ((k-2)x + m - 1)(x - 3n)^2$ $\qquad (k-2)(4) + m - 1 = 0$ $\qquad k - 2 < 0$ $\qquad 3n = -1$

$4k - 8 + m - 1 = 0$ $\qquad k < 2$ $\qquad n = -1/3$

$k = 1$

$\frac{m}{n} + k = \frac{5}{-1/3} + 1 = -14$ ۲

$4k + m = 9$

$4 + m = 9 \longrightarrow m = 5$

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۳)

$y = -1/2\, x^2 + 2x + 6$ $\qquad -1 \rightarrow 3/5$ $\qquad x = \frac{-2 \pm 4}{-1} = 6, -2$

$\Delta = 4 - 4(-3) = 16$

x		-2		6	
	$-$	0	$+$	0	$-$

$(a, b) > 7/2 \rightarrow 3/5$

$(-1, 5)$

$b - a =$

$-\frac{1}{2}x^2 + 2x + 6 > 7/2$

$5 - (-1) = 6$ ۲

$+x^2 - 4x - 5 < 0$

x		-1		5	
	$+$	0	$-$	0	$+$

$x = -1$

$x = 5$

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۴)

$f(x) = x^3 - 3x^2 - x + 3$ $\qquad x > 0$

(a, b) $\qquad \longrightarrow (x^2 - 4x + 3)(x + 1)$

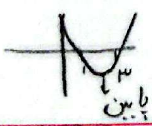
$$\begin{array}{l|l} x^3 - 3x^2 - x + 3 & x + 1 \\ \hline -x^3 - x^2 & x^2 - 4x + 3 \\ -4x^2 - x + 3 & \\ +4x^2 + 4x & \\ 3x + 3 & \\ -3x \mp 3 & \\ 0 & \end{array}$$

$(x - 3)(x - 1)(x + 1)$

$x = 3$

$x = 1$

$x = -1 \rightarrow$ غ‌ق‌ق



پایین $\rightarrow (1, 3) \longrightarrow \frac{3+1}{2} = 2$

$$\begin{array}{l|l} x^3 - 3x^2 - x + 3 & x - 1 \\ \hline -x^3 + x^2 & x^2 - 2x - 3 \\ -2x^2 - x + 3 & \\ +2x^2 - 2x & \\ -3x + 3 & \\ +3x \mp 3 & \\ 0 & \end{array}$$

$(x^2 - 2x - 3)(x - 1)$

$(x - 3)(x + 1)$

$x = 3$

$x = 1$

$x = -1$ ✗

$f(2) = 8 - 3(4) - 2 + 3$

$8 - 12 - 2 + 3 = -3$ جواب ۲

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۵)

$(a-1)x^2 + (a-1)x + 1$ $\qquad \Delta < 0$

$(a-1)^2 - 4(a-1) < 0$

a مجموعه $= (1, 5)$ $\qquad$ ۱/۵

$(a-1)(a-1-4) < 0$

$(a-1)(a-5) < 0$

$a = 1$

$a = 5$

a		1		5	
	$+$	0	$-$	0	$+$

$$\frac{m(m^2+m)}{m-2} > 0 \qquad \frac{m^2(m^2+1)}{m-2} > 0$$

$m=0^*$ $\quad$ $m=2$

$m^2=-1$ غق

$$\begin{array}{ccccc} & 0^* & & 2 & \\ - & | & - & | & + \end{array}$$

$(2,+\infty)$

ص

17

$$\frac{(x^2-x-6)(x-1)^2}{(x^2+x+1)(2-x)^3} \leq 0$$

$$\frac{(x+2)(x-3)(x-1)^2}{(x^2+x+1)(2-x)^3} \leq 0$$

$\Delta < 0$ $\quad$ $x=-2$ $\quad$ $x=3$ $\quad$ $x=1^*$ $\quad$ $x=2$

$$\begin{array}{ccccccccc} & -2 & & 1^* & & 2 & & 3 & \\ + & | & - & | & - & | & + & | & - \end{array}$$

$[-2,2)\cup[3,+\infty)$

ص

18

$$f(x)=\frac{3x^2-2x}{x^2+4} \qquad y=2$$

$$\frac{3x^2-2x}{x^2+4} < 2$$

$$\frac{3x^2-2x-2x^2-8}{x^2+4} < 0$$

$$\Delta > 0 \qquad \frac{x^2-2x-8}{x^2+4} < 0$$

$$\frac{(x-4)(x+2)}{x^2+4} < 0 \qquad x=4 \quad x=-2$$

$$\begin{array}{ccccc} & -2 & & 4 & \\ + & | & - & | & + \end{array}$$

$(-2,4)$ $\to$ (a,b)

$b-a=$

$4-(-2)=6$

ص

19

$$-1 < \frac{3x^2-4x}{x+1} < 0 \qquad \Delta=9-4(3) \qquad \Delta<0$$

$$0 < \frac{3x^2-4x}{x+1}+1 \longrightarrow 0 < \frac{3x^2-\cancel{4x}+\cancel{x}+1}{x+1} \qquad 3x^2-3x+1$$

$x+1 \to x=-1$

$$\begin{array}{ccc} & -1 & \\ - & | & + \end{array} \to (-1,+\infty)$$

$$\frac{3x^2-4x}{x+1} < 0 \qquad \frac{x(3x-4)}{x+1} < 0$$

$x=0$ $\quad$ $3x=4$ $\quad$ $x=4/3$ $\quad$ $x=-1$

$$\begin{array}{ccccccc} & -1 & & 0 & & 4/3 & \\ - & | & + & | & - & | & + \end{array}$$

$(-\infty,-1)\cup(0,\frac{4}{3})$

$\cap$

اشتراک $(0,\frac{4}{3})$

ص

20

$$\frac{x^2-10}{x} \leq 3$$

$$\frac{x^2-3x-10}{x} \leq 0 \qquad (x-5)(x+2)$$

$x=5$

$x=-2$

$x=0$

$$\begin{array}{ccccccc} & -2 & & 0 & & 5 & \\ - & | & + & | & - & | & + \end{array} \Rightarrow (-\infty,-2]\cup(0,5]$$

ص

۵

برای اینکه عبارت همواره منفی باشد $a<0$ و $\Delta<0$

$$a-1<0 \rightarrow \boxed{a<1} \quad (1)$$

$$\Delta<0 \rightarrow (a-1)^2-4(a-1)<0 \rightarrow 0<a-1<4 \rightarrow \boxed{1<a<5} \quad (2)$$

$$a-1=t \rightarrow t^2-4t<0 \rightarrow 0<t<4$$

$$1 \cap 2 = \varnothing$$