۱۴,۲۵

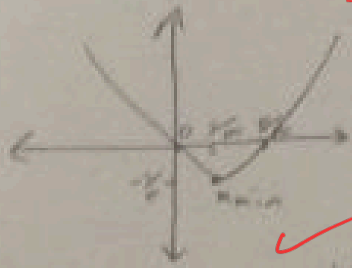
(۱

الف) $y = 3x^2 - 2x$

$S \left| \begin{array}{l} -\frac{b}{2a} = \frac{2}{6} = \frac{1}{3} \\ -\frac{1}{3} \end{array} \right.$ / $a > 0 \rightarrow min$

$3x^2 - 2x = 0 \rightarrow x(3x - 2) = 0$

$x = 0 , \frac{2}{3}$

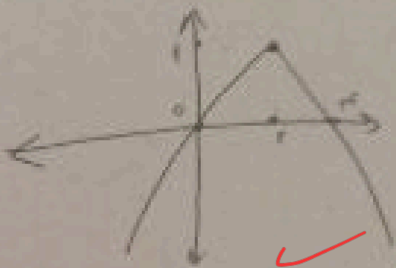


از ناحیه سوم نمی‌گذرد

ب) $y = -x^2 + 4x$

$S \left| \begin{array}{l} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ +4 \end{array} \right.$ / $a < 0 \rightarrow max$

$-x^2 + 4x = 0 \rightarrow x(-x + 4) = 0 \rightarrow x = 0 , 4$



از ناحیه سوم نمی‌گذرد

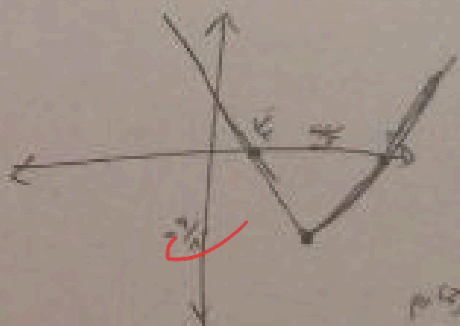
(۲

الف) $y = 2x^2 - 5x + 2$

$S \left| \begin{array}{l} -\frac{b}{2a} = \frac{5}{4} \\ -\frac{9}{8} \end{array} \right.$ / $a > 0 \rightarrow min$

$2x^2 - 5x + 2 = 0 \rightarrow (x - 2)(x - \frac{1}{2}) = 0$

$x = 2 , \frac{1}{2}$



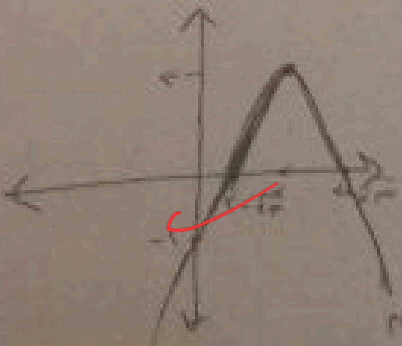
ناحیه اول و دوم و چهارم

ب) $y = -x^2 + 4x - 1$

$S \left| \begin{array}{l} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ +3 \end{array} \right.$ / $a < 0 \rightarrow max$

$-x^2 + 4x - 1 = 0 \longrightarrow \Delta = b^2 - 4ac = 16 - 4 = 12$

$x_1 , x_2 = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$



ناحیه اول و سوم و چهارم

(۳

$x^2 - x - 3 = 0$

$\Delta = 1 + 12 = 13$

$x_1 , x_2 = \frac{1 \pm \sqrt{13}}{2}$

الف) $= \frac{\alpha \beta}{\alpha - \beta} = \frac{\frac{1+\sqrt{13}}{2} \times \frac{1-\sqrt{13}}{2}}{\frac{1+\sqrt{13}}{2} - \frac{1-\sqrt{13}}{2}}$

$= \boxed{\frac{3}{\sqrt{13}}}$

۱,۵

(۹)

ب) $\alpha^2+\beta^2 = \left(\frac{1+\sqrt{13}}{2}\right)^2 + \left(\frac{1-\sqrt{13}}{2}\right)^2 = \frac{(1+\sqrt{13})^2+(1-\sqrt{13})^2}{4} =$

$\frac{1+13+2\sqrt{13}+1+13-2\sqrt{13}}{4} = \frac{28}{4} = \boxed{14}$

$\alpha^2+\beta^2 = s^2-2p = (1)^2-2(-3) = 7$

ج) $\alpha^3+\beta^3 = s^3+3sp = (1)^3+(3\times-3) = 1-9 = \boxed{-8}$

$s = \frac{1+\sqrt{13}+1-\sqrt{13}}{2} = 1$ / $p = \frac{(1+\sqrt{13})(1-\sqrt{13})}{4} = -3$

$\alpha^3+\beta^3 = s^3-3sp \rightarrow (1)^3-3(1)(-3) = 10$

د) $\alpha^3-\beta^3 = \left(\frac{1+\sqrt{13}}{2}\right)^3 - \left(\frac{1-\sqrt{13}}{2}\right)^3 = \frac{(1+\sqrt{13})^3-(1-\sqrt{13})^3}{8} =$

$\frac{1+3\sqrt{13}+3(13)+(\sqrt{13})^3 - 1 + 3\sqrt{13} - 3(13) + (\sqrt{13})^3}{8} = \frac{6\sqrt{13}+2(13\sqrt{13})}{8} = \frac{32\sqrt{13}}{8} = \boxed{4\sqrt{13}}$

(۴)

$(x-2)(x^2-ax+a)=9$

$x=2$ ... → $\Delta \le 0 \rightarrow b^2-4ac = (-a)^2-4(1)(a) = a^2-4a \le 0$

معادله‌ی x^2-ax+a می‌تواند ریشه‌ی مضاعف داشته باشد $a=2$ ← $a=4$ ← مقبول $a \in (0,4]$

$a(a-4) \le 0$ ← $\boxed{0 \le a \le 4}$

x		0		4	
y	$+$	0	$-$	0	$+$

$0<a<4$ ← $\Delta<0$ } ۲ شرط دارد

ریشه مضاعف ۲ $a=4$

اشتراک → $\boxed{0<a\le 4}$

(۵)

$2x^2-8x-a=0 \rightarrow \beta \ne \alpha$

$\alpha^2+\beta^2-4\alpha = 7$ ★

$s = \frac{-b}{a} = \frac{8}{2} = 4 \rightarrow \beta \times a = 4$

$p = \frac{c}{a} = \frac{-a}{2}$

$2x^2-8x-a=0 \xrightarrow{\div 2} x^2-4x-\frac{a}{2}=0 \rightarrow \alpha^2-4\alpha = \frac{a}{2}$

$\alpha^2+\beta^2 = s^2-2p \rightarrow \alpha^2+\beta^2 = 16+\frac{2a}{2}$

$\alpha^2+\beta^2-4\alpha = \alpha^2+\beta^2+\alpha^2-4\alpha = 16+\frac{2a}{2}+\frac{a}{2} = 7 \rightarrow 16+a = 7 \rightarrow a = -9$ (۶)

$2x^2-8x+6 = (x^2-4x+3) = (x-1)(x-3) \rightarrow x = 1, 3$

16

$ax^2 - ax - b = 0 \xrightarrow{\alpha\ \text{ریشه}} a\alpha^2 - a\alpha - b = 0 \to a(\alpha^2 - \alpha) = b \Rightarrow \alpha^2 - \alpha = \frac{b}{a}$ (9

β ریشه $\to a\beta^2 - a\beta - b = 0 \to a(\beta^2 - \beta) = b$ $\qquad \boxed{\alpha^2 = \frac{b}{a} + \alpha}$

$\beta^2 - \beta = \frac{b}{a} \to \boxed{\beta^2 = \frac{b}{a} + \beta}$

$40\beta^2 + 20\alpha^2 - 20\beta = 17 \xrightarrow{\beta^2, \alpha^2\ \text{جایگذاری}} 40\left(\beta + \frac{b}{a}\right) + 20\left(\alpha + \frac{b}{a}\right) - 20\beta = 17$

$S = \frac{-b}{a} = \frac{+a}{a} = 1$

$40\beta + \frac{40b}{a} + 20\alpha + \frac{20b}{a} - 20\beta = 17$

$20\beta + \frac{60b}{a} + 20\alpha = 17 \to 20(\beta + \alpha) + \frac{60b}{a} = 17$

$\frac{60b}{a} = -3 \to \boxed{\frac{b}{a} = -\frac{1}{20}}$

$P = \frac{c}{a} = \frac{-b}{a} = \frac{1}{20}$

$|\alpha - \beta| = \sqrt{\alpha^2 + \beta^2 - 2\alpha\beta}$, $\quad \alpha^2 + \beta^2 = 1 - 2\alpha\beta$

$\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2 = 1$

$\alpha^2 + \beta^2 = 1 - 2\alpha\beta$

$\sqrt{1 - 4 \times \frac{1}{20}} = \sqrt{1 - \frac{4}{20}}$

$\sqrt{\frac{10}{10} - \frac{2}{10}} = \sqrt{\frac{8}{10}} = \boxed{\frac{\sqrt{8}}{\sqrt{10}}} = \frac{\sqrt{80}}{10}$

(8

$y = a(x-h)^2 + k \to k = -\frac{1}{4} \to a(x-h)^2 - \frac{1}{4}$

$(1, \beta) \to \beta = a(1-h)^2 - \frac{1}{4}$

$(-5, \beta) \to \beta = a(-5-h)^2 - \frac{1}{4}$

$\to a(1-h)^2 - \frac{1}{4} = a(-5-h)^2 - \frac{1}{4}$

$1 - h = \pm(-5 - h)$

$\to 1 - h = -5 - h$ غ ق

$\to 1 - h = 5 + h \Rightarrow \underline{h = -2}$

$(0, \frac{3}{4}) \Rightarrow \frac{3}{4} = a(0-h)^2 - \frac{1}{4} = \frac{3}{4} = a(-2)^2 - \frac{1}{4} \to \frac{3}{4} + \frac{1}{4} = 4a \to \boxed{a = \frac{1}{4}}$

$\beta = \frac{1}{4}(1+2)^2 - \frac{1}{4} \Rightarrow \beta = \frac{9}{4} - \frac{1}{4} = \frac{8}{4} = \boxed{2}$

$x^2+4x+5=0$ $\quad S=\frac{-b}{a}=\frac{-4}{1}=-4$ / $P=\frac{c}{a}=\frac{5}{1}=5$ (9)

$\alpha^2+\beta^2=S^2-2P \to$ [illegible] $\to \beta=-4-\alpha$

[illegible]

$x_1, x_2=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{-2\pm\sqrt{[illegible]}}{10}$ [illegible] ✗ 1/5

$P=\frac{c}{a}=\frac{1}{24}$ $\qquad S=\frac{-b}{a}=\frac{m+14}{24}$

$24x^2-(m+14)x+1=0$ (10

$\sqrt{\frac{1}{\alpha}}+\sqrt{\frac{1}{\beta}}=[illegible] \to \frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}}=[illegible] \to \sqrt{\alpha}+\sqrt{\beta}=[illegible]$ $)(\ )^2$

$\alpha+\beta+2\sqrt{\alpha\beta}=\frac{[illegible]}{24}$

$\alpha+\beta+\frac{2}{\ [illegible]}=\frac{[illegible]}{24}$

$\to \frac{m+14}{24}=$ [illegible] $\to m+14=13 \to \boxed{m=-1}$

$3x^2+2x+2=0 \to -x^2+2x+2=0$

$P=\frac{c}{a}=\frac{2}{-1}=\boxed{-2}$ ✓ 2

۶

$x_S = \frac{۷-۲a+۲a+۳}{۲} = ۵ \rightsquigarrow y_S = ۳$

$\begin{cases} ۷-۲a>۰ \\ ۲a+۳>۰ \\ a-۲>۰ \end{cases} \rightsquigarrow \underbrace{۲<a<۳٫۵}_{a=۳}$

نقاط A و B با طول و عرض صحیح هستند ←

$a=۳ \begin{cases} A(۹,۱) \\ B(۱,۱) \end{cases} \rightsquigarrow y-۳ = a(x-۵)^۲ \xrightarrow{(۱,۱)} a = -\frac{۱}{۸}$

$(y-۳) = -\frac{۱}{۸}(۰-۵)^۲ \longrightarrow y = ۳ - \frac{۲۵}{۸} = -\frac{۱}{۸}$

فاصله تا مبدا مختصات $\frac{۱}{۸}$ است

۷

$ax^۲ - ax - b = ۰ \longrightarrow S = \frac{a}{a} = ۱ \rightsquigarrow \alpha+\beta = ۱ \rightsquigarrow \alpha = ۱-\beta$

$۴۰\beta^۲ + ۲۰(۱-\beta)^۲ - ۲۰\beta = ۱۷ \rightsquigarrow ۶۰\beta^۲ - ۶۰\beta + ۳ = ۰ \rightsquigarrow \beta = \frac{۲۰ \pm \sqrt{۴۰۰-۸۰}}{۴۰}$

$\alpha-\beta = ۱-۲\beta = ۱-۲\left(\frac{۱۰\pm\sqrt{۸۰}}{۲۰}\right) = ۱-\left(۱\pm\frac{۲}{\sqrt{۵}}\right) = \frac{\pm ۲}{\sqrt{۵}}$

$\alpha-\beta = \frac{۲}{\sqrt{۵}}$ ← اختلاف همیشه مثبت اعلام می‌شود پس

۹

$۳\alpha^۲+۲\beta^۲ = \frac{۵}{۲}(\alpha^۲+\beta^۲) + \frac{۱}{۲}(\alpha^۲-\beta^۲) = ۱۲\sqrt{۲}+۸۵$

$\frac{۵}{۲}(S^۲-۲P) + \frac{۱}{۲}(S)\left(\frac{\sqrt{\Delta}}{|a|}\right) = ۱۲\sqrt{۲}+۸۵$

$\frac{۵}{۲}(۳۶-۲a) + \frac{۱}{۲}(-۶)(-\sqrt{۳۶-۴a}) = ۱۲\sqrt{۲}+۸۵$

$۹۰ - ۵a + ۳\sqrt{۳۶-۴a} = ۱۲\sqrt{۲}+۸۵ \rightarrow ۹۰-۵a = ۸۵ \longrightarrow a=۱$