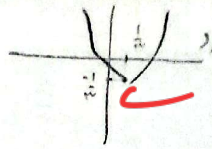
۱. دیانا وطن خواه

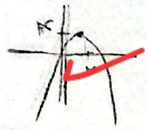
۱۹/۷۵ آفرین

۱) الف) $y = 3x^2 - 2x$ $\left| \frac{-b}{2a} = \frac{2}{6} = \frac{1}{3} \right.$ $y = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3}$



از ناحیه ۳ عبور نمی‌کند ✓

ب) $y = -x^2 + 4x$ $\left| \frac{-b}{2a} = 2 \right.$ $y = 4$



از ناحیه ۳ عبور نمی‌کند ✓ (۲)

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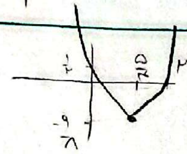
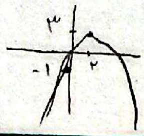
۲. الف) $x = \frac{-b}{2a} = \frac{5}{4}$ $y = \frac{-\Delta}{4a} = -\frac{9}{8}$

ریشه‌ها: $\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{5 \pm \sqrt{9}}{4}$ → $\frac{8}{4} = 2$ ، $\frac{2}{4} = \frac{1}{2}$



از ناحیه ۱ و ۲ و ۴

ب) $\left| x = \frac{-b}{2a} = 2 \right.$ $y = 3$



از ۱ و ۳ و ۴ ✓ (۲)

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۳. الف) $\frac{\alpha + \beta}{\alpha - \beta}$ ، $\alpha + \beta \to S = -\frac{b}{a} = 1$

$\alpha - \beta \to \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{1 - 4(-3)(1)}}{1} = \sqrt{13} \Rightarrow \frac{1}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

ب) $\alpha^2 + \beta^2 = S^2 - 2P = 1 - 2(-3) = 7$ ✓

ج) $\alpha^3 + \beta^3 = S^3 - 3PS = 1 - 3(-3)(1) = 10$ ✓

$(\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta) \to (\sqrt{13})(1 + 6 - 3) = 4\sqrt{13}$

د) $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta) = (\sqrt{13})^3 + 3(-3)(\sqrt{13}) \Rightarrow 13\sqrt{13} - 9\sqrt{13} = -4\sqrt{13}$

(صفر ۱)

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۴. $y = (x - 2)(x^2 - ax + a)$

$x - 2 = 0$ ، $x = 2$

$\Delta < 0 \to a^2 - 4a < 0 \to a(a - 4) < 0$ ، $0 < a < 4$

+ | − | + (۰ ، ۴)

$x^2 - ax + a \xrightarrow{x=2} 4 - 2a + a = 0$ ، $-a = -4$ ، $a = 4$

$0 < a \leq 4$ ✓ (۲)

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۵. $3x^2 - 12x - a = 0$

$2\alpha^2 + \beta^2 - 4\alpha = 7$

$\beta^2 + \alpha^2 + \alpha^2 - 4\alpha = 7$

$4\alpha + \frac{a}{3}$

$3x^2 - 12x + 9 \Rightarrow$ ، $3 - 12 + 9 = 0$ ، $x_1 = 1$ ، $x_2 = \frac{c}{a} = \frac{9}{3} = 3$

$\frac{-9}{3} = -3$ برابر

$\alpha^2 + \beta^2 = 16 + \frac{2a}{3} \Rightarrow S^2 - 2P$

$16 + \frac{2a}{3} \to 4\alpha - 4\alpha + \frac{a}{3} = 7$

$\to \frac{3a}{3} = 9$

(۲)

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$A = (2a+3, \frac{a-2}{\text{برابر}})$, $B(7-2a, \frac{a-2}{\text{برابر}})$

$x_{\text{راس}} = \frac{2x+3+7-2a}{2} = 5$ $\left| \begin{matrix} 5 \\ 3 \end{matrix} \right.$

$b = 5$ $b - 2 = 3$

$y = a(x-5)^2 + 3$ $A(1,1)$

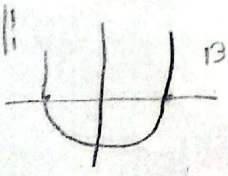
$1 = 16a + 3$

$a = -\frac{1}{8}$

$y = 0 = -\frac{1}{8}(x-5)^2 + 3$

$y(0) = -\frac{25}{8} + 3 = -\frac{1}{8}$ → عرض از مبدا $\frac{1}{8}$

$B(9,1)$



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$\alpha x^2 - \alpha x - b = 0$ $\alpha + \beta = 1$

$\beta^2 = (1-\alpha)^2$

$4\beta^2 + 2\alpha^2 - 2\beta = 17$

$4(1-\alpha)^2 + 2(\alpha^2) - 2(1-\alpha) = 17 \rightarrow 6\alpha^2 - 6\alpha + 30 = 0$

$s = 1$

$p = \frac{1}{2}$

$|\alpha - \beta| = \sqrt{s^2 - 4p} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

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$(-5,\beta), (1,\beta) \rightarrow \frac{-5+1}{2} = -2 \rightarrow x = -2$ $ext \left| \begin{matrix} -2 \\ -\frac{1}{2} \end{matrix} \right. \rightarrow \frac{-b}{2a} = -2$

$b = 4a$

$\frac{-\Delta}{4a} = \frac{-b^2 + 4ac}{b} = -b + \not{c}\,\frac{3}{2} = -\frac{1}{2} \rightarrow b = 2$

$\frac{1}{2}x^2 + 2x + \frac{3}{2} \xrightarrow{x=1} \frac{1}{2} + 2 + \frac{3}{2} = b = 4$

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$x^2 + 6x + a = 0 \begin{cases} \alpha = -3 + \sqrt{9-a} \Rightarrow \alpha^2 = 18 - a - 6\sqrt{9-a} \\ \beta = -3 - \sqrt{9-a} \rightarrow \beta^2 = 18 - a + 6\sqrt{9-a} \end{cases}$

$3\beta^2 + 2\beta^2 = 90 - 5a - 6\sqrt{9-a} = 12\sqrt{2} - 85$

$5a + 6\sqrt{9-a} = 5 + 6\sqrt{8} \rightarrow a = 1$

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$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5 \longrightarrow \frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} = 5$

$\alpha \cdot \beta = \frac{1}{36}$, $\alpha + \beta = \frac{m+14}{36}$ $(\sqrt{\alpha} + \sqrt{\beta})^2 = \alpha + \beta + 2\sqrt{\alpha\beta}$

$\frac{m+14}{36} + \frac{2}{6} = \frac{m+26}{36}$

$\frac{6 \times \sqrt{m+26}}{6} = 5$

$\frac{\sqrt{m+26}}{6} = \sqrt{\alpha} + \sqrt{\beta}$

$m + 26 = 25 \rightarrow m = -1$

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$\alpha_2 \cdot \beta_2 \rightarrow mx^2 + 3x + 2 \longrightarrow \frac{c}{a} = \frac{2}{m} = -2$