سامیار داودی — تکلیف ۲۴ — دهم دختر B

۱) الف)

$y = 2x^2 - 6x + 1$

$ext \begin{cases} \frac{-b}{2a} \Rightarrow \frac{6}{4} = 1 \\ \frac{-\Delta}{4a} \Rightarrow \frac{4ac - b^2}{4a} \Rightarrow \frac{8 - 16}{8} = \frac{-8}{8} = -1 \end{cases}$ $\quad ext \begin{vmatrix} 1 \\ -1 \end{vmatrix}$

ب)

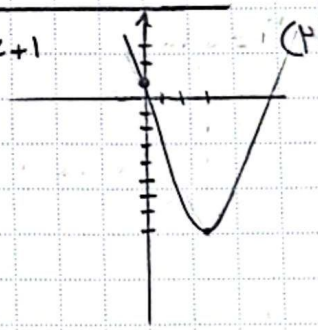
$y = -2x^2 + 3x - 5$

$ext \begin{cases} \frac{-b}{2a} \Rightarrow \frac{-3}{-4} = \frac{3}{4} \\ \frac{-\Delta}{4a} \Rightarrow \frac{4ac - b^2}{4a} \Rightarrow \frac{40 - 9}{-8} = \frac{-31}{8} \end{cases}$ $\quad ext \begin{vmatrix} \frac{3}{4} \\ \frac{-31}{8} \end{vmatrix}$

۲) الف) $y = x^2 - 4x + 1$

$a > 0 \rightarrow min$

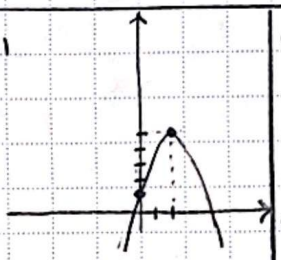
$ext \begin{vmatrix} 2 \\ -3 \end{vmatrix}$



ب) $y = -x^2 + 6x + 1$

$a < 0 \rightarrow max$

$ext \begin{vmatrix} 3 \\ 10 \end{vmatrix}$



۳) $4x^3 + kx^2 - 9x - 2 = 0$

$\alpha + \beta + \theta = \frac{-k}{4}$

$\alpha\beta + \beta\theta + \alpha\theta = \frac{9}{4}$

$\alpha\beta\theta = \frac{1}{2}$

$\rightarrow \alpha + \beta = 1 \rightarrow \theta = \frac{-k}{4} - 1$

$\rightarrow \theta = \frac{17}{4}$

$-2 + \alpha\theta + \beta\theta = \frac{9}{4} \rightarrow \alpha\theta + \beta\theta = \frac{17}{4} \rightarrow \theta(\underbrace{\alpha + \beta}_{1}) = \frac{17}{4}$

$\longrightarrow \theta = \frac{-k}{4} - 1 = \frac{17}{4} \rightarrow \frac{-k}{4} = \frac{21}{4} \rightarrow \boxed{k = -21}$

۴)

α, β ریشه‌ها

$x^2 - 3mx + m = 0$

$ab = 3m$

$p = m$

$|\sqrt{\alpha} - \sqrt{\beta}| = 1 \xrightarrow{\text{به توان ۲}} \alpha + \beta - 2\sqrt{\alpha\beta} = 1$

$\rightarrow 3m - 2\sqrt{m} = 1 \xrightarrow{\sqrt{m} = t} 3t^2 - 2t - 1 = 0 \xrightarrow[\text{ولتا به دست آوردم}]{\text{ریشه‌ها را با}} t \swarrow \begin{matrix} 1 \\ -\frac{1}{3} \end{matrix}$

$\rightarrow m = t^2 \rightarrow m \swarrow \begin{matrix} 1 \\ \frac{1}{9} \end{matrix}$

$2x^2 - mx - m = 0$

$p = \frac{-m}{2} \swarrow \begin{matrix} \frac{-1}{2} \\ \frac{-1}{18} \end{matrix} \rightarrow$ حاصل ضرب ریشه‌ها $= \frac{-1}{2}, \frac{-1}{18}$

۵)

$y = 2x^2 - (m+2)x + m$

صفرهای تابع $\rightarrow x_1, x_2$

نقطه تقاطع با محور yها $\rightarrow x = 0 \rightarrow y = m$

$\rightarrow$ سه رأس مثلث $\rightarrow (x_1, 0), (x_2, 0), (0, m)$

$(\text{مساحت}) \mathcal{A} = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$

$\rightarrow \mathcal{A} = \frac{1}{2}\left| x_1(0 - m) + x_2(m - 0) + 0(0 - 0) \right| = \frac{1}{2}\left| -mx_1 + mx_2 \right|$ ★

$\mathcal{A} = \frac{3}{4}$

برای این معادلهٔ درجه ۲ $x^2 - \frac{m+2}{2}x + \frac{m}{2} = 0 \rightarrow x_2 - x_1 = \sqrt{(x_1 + x_2)^2 - 4x_1x_2}$

$(\text{مجموع}) \mathcal{A} = \frac{m+2}{2}$

$p = \frac{m}{2}$

$\rightarrow x_2 - x_1 = \sqrt{(x_1 + x_2)^2 - 4x_1x_2} = \sqrt{\left(\frac{m+2}{2}\right)^2 - \frac{4m}{2}} =$

$\rightarrow \sqrt{\frac{(m+2)^2}{4} - 2m} = \sqrt{\frac{m^2 + 4m + 4 - 8m}{4}} = \sqrt{\frac{m^2 - 4m + 4}{4}} = \sqrt{\frac{(m-2)^2}{4}} =$

$\rightarrow \frac{|m-2|}{2}$

اگر -3 باشد معادلهٔ درجه ۲ غ ق ق است.

ادامه ★ $|m(x_2 - x_1)| = \frac{3}{2} \rightarrow \left| m \cdot \frac{|m-2|}{2} \right| = \frac{3}{2} \rightarrow |m(m-2)| = 3 \rightarrow m(m-2) = \pm 3$

$\rightarrow m^2 - 2m - 3 = 0 \rightarrow m \swarrow \begin{matrix} 3 \\ -1 \end{matrix}$

(ادامه صفحه بعد)

ادامه ۵)

$y = x^2 - mx + 1$

$\rightarrow \frac{-b}{2a} = \frac{m}{2} = x_s \rightarrow x_s \swarrow \frac{3}{2} \searrow -\frac{1}{2}$

$\boxed{-\frac{1}{2} \text{ و } \frac{3}{2}}$

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۶)

$y = ax^2 + 3x + a$

$y_{min} = \frac{-\Delta}{4a} = \frac{4ac - b^2}{4a} = \frac{4a^2 - 9}{4a} = \frac{V}{8} \rightarrow 32a^2 - V2 = 28a$

$\rightarrow 32a^2 - 28a - V2 = 0 \rightarrow 8a^2 - Va - 18 = 0$

$\rightarrow a = \frac{V \pm \sqrt{(-V)^2 - 4(8)(-18)}}{16} = \frac{V \pm \sqrt{925}}{16} = \frac{V \pm 25}{16}$ ریشه‌ها

$\rightarrow a \swarrow \frac{-9}{8} \quad \searrow 2 \rightarrow \boxed{a = 2 \text{ , } a = -\frac{9}{8}} \rightarrow$ ۲ مقدار

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۷)

$x^2 - (a+1)x + a = 0$

ریشه‌ها $x_1 = 2n - 1$, $x_2 = 2n + 1$ $\quad n \in \mathbb{N} \rightarrow x_1 + x_2 = 2n - 1 + 2n + 1 = 4n$

$x_1 x_2 = (2n-1)(2n+1) = 4n^2 - 1$

$\rightarrow x_1 + x_2 = a + 1 = 4n \rightarrow a = 4n - 1$

$x_1 x_2 = a \rightarrow a = 4n^2 - 1$

$\rightarrow 4n^2 - 1 = 4n - 1 \rightarrow 4n^2 - 4n = 0 \rightarrow 4n(n-1) = 0$

$\rightarrow n \swarrow 0 \quad \searrow 1$ (!) $\rightarrow$ چون ریشه‌ها طبیعی‌اند $\rightarrow x_1 = 1$, $x_2 = 3$

$a = 4n - 1 = (4 \times 1) - 1 = 3$

$x^2 - (3a+1)x + b = 0 \rightarrow x^2 - 10x + b = 0$

ریشه‌ها $z_1 = 2m$, $z_2 = 2m + 2$ $\quad m \in \mathbb{N} \rightarrow z_1 + z_2 = 4m + 2$

$z_1 z_2 = 4m(m+1)$

$\rightarrow z_1 + z_2 = 3a + 1 \rightarrow 4m + 2 = (3 \times 3) + 1 = 10 \rightarrow 4m + 2 = 10 \rightarrow m = 2$

$\rightarrow z_1 = 4$, $z_2 = 6$

$\rightarrow b = z_1 z_2 = 24$

$\boxed{z_1 z_2 - x_1 x_2 = 24 - 3 = 21}$

۸)

$y = -ax^2 + ax + 2$

$x_s = \frac{-b}{2a} = \frac{-a}{-2a} = \frac{1}{2}$

$y_s = \frac{4ac - b^2}{4a} = \frac{-8a - a^2}{-4a} = \frac{a}{4} + 2$

$b - a = (-4) - (-12)$

$b - a = 4$

$y = 2bx^2 - bx - 1$

$y = \frac{a}{4} + 2 \quad x = \frac{1}{2}$

$\rightarrow \frac{a}{4} + 2 = 2b\left(\frac{1}{2}\right)^2 - b\left(\frac{1}{2}\right) - 1 \rightarrow \frac{a}{4} + 2 = \frac{b}{2} - \frac{b}{2} - 1 \rightarrow \frac{a}{4} + 2 = -1 \rightarrow a = -12$

$x_s = \frac{-b}{2a} = \frac{b}{4b} = \frac{1}{4}$

$y_s = -\frac{b}{8} - 1$

جایگذاری در تابع اول

$\frac{-b}{8} - 1 = \frac{3}{4} - 3 + 2 \rightarrow \frac{-b}{8} - 1 = \frac{3}{4} - 1 \rightarrow \frac{-b}{8} = \frac{3}{4} \Rightarrow -b = 6 \rightarrow b = -6$

۹)

$y = 2\Delta\alpha x^2 + 4x + \beta$

$\alpha < \beta$

رابع دوم یا چهارم

$x_s = \frac{-b}{2a} = \frac{-4}{10\alpha} = \frac{-2}{2\Delta\alpha}$

$y_s = 2\Delta\alpha\left(\frac{-2}{2\Delta\alpha}\right) + 4\left(\frac{-2}{2\Delta\alpha}\right) + \beta = \frac{4}{2\Delta\alpha} - \frac{8}{2\Delta} + \beta$

ریشه‌ها: $x_1 = \alpha \quad x_2 = \beta$

$x_1 + x_2 = \alpha + \beta = -\frac{b}{a} = \frac{-4}{2\Delta\alpha}$

$x_1 x_2 = \alpha\beta = \frac{c}{a} = \frac{\beta}{2\Delta\alpha}$

$\alpha < 0, \beta < 0$
راس در ربع ۳

$\alpha > 0, \beta > 0$
راس در ربع ۲

$\alpha > 0 \rightarrow \min$، کمترین مقدار y_s، بیشترین x_s

$\alpha < 0 \rightarrow \max$، بیشترین مقدار y_s، کمترین x_s

$\rightarrow x_s = \frac{-2}{2\Delta\alpha}$

$\alpha > 0 \rightarrow x_s < 0$

$\alpha < 0 \rightarrow x_s > 0$

$\beta < \alpha < 0 \rightarrow \alpha < 0 \rightarrow \frac{4}{2\Delta\alpha} < 0$

$\rightarrow y_s < 0$

(۱۰ $x^2-(a^2+b^2-12)x+(a+b-1)=0$

$x_1+x_2=a^2+b^2-12$

$x_1x_2=a+b-1$

$a+b=Q \rightarrow x_1x_2=Q-1$ ، $a^2+b^2=Q^2-2ab \rightarrow$

$x_1+x_2=Q^2-2ab-12$

$\rightarrow x_1=1$ ، $x_2=?$

$x_1x_2=Q-1 \rightarrow x_2=Q-1 \rightarrow x_1+x_2=1+(Q-1)=Q$

$\rightarrow x_1+x_2=a^2+b^2-12=Q^2-2ab-12$

$\rightarrow Q=Q^2-2ab-12 \rightarrow Q^2-2ab-Q-12=0 \rightarrow 2ab=Q^2-Q-12$

$\rightarrow ab=\frac{Q^2-Q-12}{2}$

$Q^2-Q-12=$ صحیح $\rightarrow 3^2-3-12=18 \rightarrow ab=9 \rightarrow a+b=6$

$x^2-(a^2+b^2-12)x+a+b-1=x^2-(9+9-12)x+6-1=x^2-6x+5$

دسته‌ها ۱، ۵ ✓ طبیعی

پس $\boxed{a+b=6}$