۱. دیانا وطن خواه

الف) $x = \frac{-b}{2a} = \frac{-(-4)}{4} = 1$ نوع سهمی: min

$y = \frac{-\Delta}{4a} = \Delta = 14 - 4(2) = 8 \rightarrow \frac{-8}{8} = -1$ راس $(1, -1)$

ب) $x = \frac{-3}{-4} = \frac{3}{4}$ راس $(\frac{3}{4}, \frac{-31}{8})$

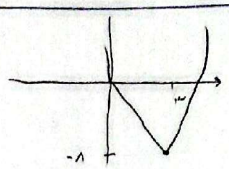
نوع سهمی: max

$y = 9 - 4(10) \Rightarrow 9 - 40 = -31 \rightarrow \frac{-31}{8}$

۲.

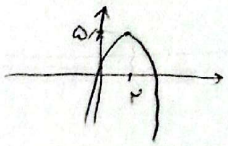
الف) $x = \frac{4}{2} = 2$ راس $(2, -1)$

$y = 9 - 12 + 1 = -1$



ب) $x = \frac{-4}{-2} = 2$ راس $(2, 5)$

$y = -4 + 4 + 1 = 5$



۳. $4(2^3) + 4(12) - 18 - 2 = 0 \rightarrow 4k = -12 \quad k = -3$

$4(-1)^3 + k + 9 - 2 = 0 \longrightarrow 12 = -3$

$4\alpha^3 + 4\alpha - 9\alpha - 2 = 0$

$4\beta^3 + 4\beta - 9\beta - 2 = 0$

$\alpha + \beta = 1 \quad x^2 - 5x + P = 0$

$\alpha\beta = -2 \quad x^2 - 1x - 2 = 0$

$(x-2)(x+1) = 0 \quad x = -1, \; x = 2$

۴. $2m^2 + mx - m = 0$

$\sqrt{\alpha} - \sqrt{\beta} = 1$

$S = \frac{-b}{a} = \frac{-m}{1} = -m$

$(\sqrt{\alpha} - \sqrt{\beta})^2 = \frac{\alpha + \beta}{2m} - 2\sqrt{\frac{\alpha\beta}{m}} = 1$

$4m - 2m = 1$

$m = 1$ جایگذاری $\longrightarrow 2m^2 + x - 1$

$x^2 + \frac{x}{2} - \frac{1}{2} \longrightarrow P = -\frac{1}{2}$

۵. $2x^2 - (m+2)x + m = 0 \quad \begin{cases} x_1 + x_2 = \frac{m+2}{2} \\ x_1 x_2 = \frac{m}{2} \end{cases}$ $A(x_1, 0) \; / \; B(x_2, 0) \; / \; C(0, m)$

$S = \frac{1}{2} |x_1 - x_2| \times |m| = 4$

① $m > 2 \Rightarrow m(m-2) = 3 \Rightarrow m^2 - 2m - 3 = 0 \Rightarrow (m-3)(m+1) = 0 \xrightarrow{m>2} m = 3 \quad m = -1$

② $0 < m < 2 \Rightarrow m(2-m) = 3 \Rightarrow m^2 - 2m + 3 = 0 \longrightarrow \Delta < 0$ ③ $m < 0 \rightarrow (-m)(2-m) = 3 \rightarrow m^2 - 2m - 3 = 0 \xrightarrow{m<0}$

$m = 3$ یا $m = -1 \quad y = -x^2 - mx + 1$ راس $x = \frac{m}{2}$ راس $y = (\frac{m}{2})^2 - m(\frac{m}{2}) + 1 = 1 - \frac{m^2}{4}$

① $m = 3 \Rightarrow$ راس $y = 1 - \frac{9}{4} = -\frac{5}{4}$ ② $m = -1$ راس $y \rightarrow = 1 - \frac{1}{4} = \frac{3}{4}$

$\frac{-5}{4a}=\frac{7}{8}$ $\qquad$ $\frac{-(9-4(a)(a))}{4a}=\frac{7}{8}$

$\frac{-9+4a^2}{\frac{4a^2}{a}}=\frac{7}{8}$ $\qquad$ $8a^2-18=7a$ $\qquad$ $8a^2-7a-18=0$

$P<0$ یک جواب منفی یک جواب مثبت $\qquad$ $a>0$ دو ریشه حقیقی فقط a

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$x^2-(a+1)x+a=0$

$1-a-1+a=0$

$x_1=1$

$\frac{c}{a}=x_2=a\to 3$ $\longrightarrow$ $x^2-(3\times 3+1)x+b=$

$x^2-10x+b=0$ $\qquad$ $y=4$

$S=10$ $\qquad$ $y=9$

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$y=-ax^2+ax+2$ $\qquad$ $y_{x_v}=2b(x_v)^2-b(x_v)-1=\,?$

$\frac{a}{2a}=\frac{1}{2}$ $\qquad$ $y=-a(\frac{1}{2})^2+a(\frac{1}{2})+2=\frac{a}{4}+2$ $\qquad$ $\frac{a}{4}+2=2b(\frac{1}{4})^2-b(\frac{1}{4})-1\Rightarrow\frac{a}{4}+2=-1$

$\frac{a}{4}=-1-2\to a=-12$

$x_v=-\frac{-b}{4b}=\frac{1}{4}$

$y_v=2b(\frac{1}{4})^2-b(\frac{1}{4})-1\Rightarrow y_v=-\frac{b}{8}-1=-12(\frac{1}{4})^2+(-12)(\frac{1}{4})+2\to b=-9$

$b-a=(-9)-(-12)=3$

$\alpha+\beta=-\frac{4}{25\alpha}$ $\longrightarrow$ $\alpha(\alpha+\beta)=-\frac{4}{25}\Rightarrow\alpha^2+\alpha\beta=-\frac{4}{25}$ $\qquad$ $\alpha\beta=\frac{\beta}{25\alpha^2}\Rightarrow\beta=\alpha=\frac{1}{25\alpha}\Rightarrow$

$\alpha\beta=\frac{\beta}{25\alpha\frac{1}{5}}=\frac{\beta}{5}$ $\qquad$ ① $\alpha^2+\alpha\beta=-\frac{4}{25}\longrightarrow(\frac{1}{5})^2+\frac{\beta}{5}=-\frac{4}{25}\Rightarrow\beta=-1,\ \alpha=\frac{1}{5}$ $\qquad$ $25\alpha^2=1\to\alpha=\pm\frac{1}{5}$

② $\alpha^2+\alpha\beta=-\frac{4}{25}\longrightarrow(-\frac{1}{5})^2+(-\frac{\beta}{5})=-\frac{4}{25}\Rightarrow\beta=1\ \ \alpha=-\frac{1}{5}$ $\qquad$ $\beta>\alpha$ ✗ $\qquad$ $\beta>\alpha$ ✓

$x_v=\frac{-b}{2a}=\frac{2}{5}$

$y_v=-5(\frac{2}{5})^2+4(\frac{2}{5})+1\Rightarrow y_v=\frac{9}{5}$ $\qquad$ $V=(\frac{2}{5},\frac{9}{5})$ ناحیه اول

$a+b=-\frac{-(a^2+b^2-12)}{1}\Rightarrow a+b=a^2+b^2-12$ ① $\qquad$ $a\times b=\frac{a+b-1}{1}\Rightarrow a\times b=a+b-1$ ②

② $a+b=ab+1$ $\qquad$ $a^2+b^2=(a+b)^2-2ab$ $\xrightarrow{①}$ $a+b=[(a+b)^2-2ab]-12$ ②

$[ab+1]=[(ab+1)^2-2ab]-12$ $\qquad$ ② $S=P+1\to P=S-1$ $\qquad$ ① $S=(a^2+b^2)-12$, $\qquad$ $a^2+b^2=S^2-2P$

$S=(S^2-2P)-12\xrightarrow{P=S-1}S(S^2-2(S-1))-12\Rightarrow S=S^2-2S-10=0\Rightarrow$

$(S-5)(S+2)=0$

$S=5$ یا $S=-2$ $\longrightarrow$ $S=a+b>2\Rightarrow S=-2$ ✗ $\qquad$ $S=5$ ✓