نام و نام خانوادگی: تانیا ترک ترابی سری: ۲۴ گروه: دهم یکشنبه

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الف) $y = 2x^2 - 4x + 1$ $\min \begin{cases} \frac{-b}{2a} = \frac{-(-4)}{4} = 1 \\ y = 2 - 4 + 1 = -1 \end{cases}$

ب) $y = -2x^2 + 3x - 5$ $\max \begin{cases} -\frac{b}{2a} = -\frac{3}{-4} = \frac{3}{4} \\ y = -2(\frac{3}{4}) + 3(\frac{3}{4}) - 5 = -\frac{3}{2} + \frac{9}{4} - 5 = -\frac{23}{4} \end{cases}$

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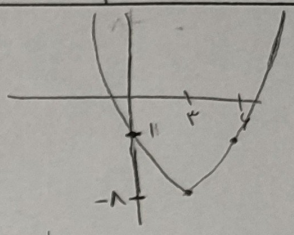
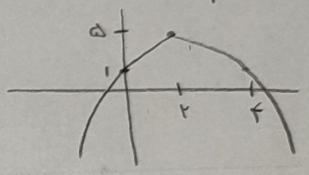
الف) $y = x^2 - 4x + 1$ $\min \begin{cases} -\frac{b}{2a} = 2 \\ y = 4 - 8 + 1 = -3 \end{cases}$

x	0	2	4
y	1	-3	1



ب) $y = -x^2 + 4x + 1$ $\max \begin{cases} -\frac{b}{2a} = 2 \\ y = -4 + 8 + 1 = 5 \end{cases}$

x	0	2	4
y	1	5	1



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$4x^3 + 8x^2 - 9x - 2 = 0 \Rightarrow \underbrace{(x-\alpha)(x-\beta)}_{x^2 - (\alpha+\beta)x + \alpha\beta = x^2 - x - 2}(4x+1) \Rightarrow (x^2 - x - 2)(4x+1) \Rightarrow$

$(x^2)(4x+1) - (x)(4x+1) - 2(4x+1) = 4x^3 + (1-4)x^2 + (-1-8)x - 2 = 4x^3 + 8x^2 - 9x - 2$

$8 = 1 - 4$ ، $-9 = -1 - 8$ ، $-2 = -2 \Rightarrow 1 = 1$ $8 = \underbrace{1}_{1} - 4 \Rightarrow 8 = -3$

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$x^2 - 2mx + m^2 - 6$ $|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 1 \Rightarrow (1)^2 = \left(\sqrt{9m^2 - 4m}\right)^2 \Rightarrow 9m^2 - 4m - 1 = 0$

$m = \frac{4 \pm \sqrt{16 + 36}}{18} = \frac{2 \pm \sqrt{13}}{9}$

$\alpha \times \beta = \frac{c}{a} = m$ ، $\alpha + \beta = -\frac{b}{a} = 2m$

$2x^2 - mx - m = 0$

$\alpha \times \beta = \frac{c}{a} = -\frac{m}{2} \Rightarrow -\frac{2 + \sqrt{13}}{8}$ یا $-\frac{2 - \sqrt{13}}{8}$

نقاط روی مثلث $\leftarrow A(x_1, 0) / B(x_2, 0) / C(0, m)$

$2x^2 - (m+2)x + m = 0$ $\begin{cases} x_1 + x_2 = \frac{m+2}{2} \\ x_1 x_2 = \frac{m}{2} \end{cases}$

$S = \frac{1}{2}|x_1 - x_2| \times |m| = \frac{3}{4}$ $|x_1 - x_2| = \frac{\sqrt{(m+2)^2 - 8m}}{2} = \frac{|m-2|}{2}$

$\frac{1}{2} \times |m| \times \frac{|m-2|}{2} = \frac{3}{4} \Rightarrow |m| \times |m-2| = 3$

① $m > 2 \Rightarrow m(m-2) = 3 \Rightarrow m^2 - 2m - 3 = 0 \Rightarrow (m-3)(m+1) = 0 \xrightarrow{m>2} \boxed{m = 3}$

② $0 < m < 2 \Rightarrow m(2-m) = 3 \Rightarrow m^2 - 2m + 3 = 0 \rightarrow \Delta < 0$

③ $m < 0 \Rightarrow (-m)(2-m) = 3 \Rightarrow m^2 - 2m - 3 = 0 \Rightarrow (m-3)(m+1) = 0 \xrightarrow{m<0} m = -1$

ادامه سوال در صفحه بعد است

$m=2$ یا $m=-1$ $\quad y=-x^2-mx+1$ $\quad$ طول رأس $=\frac{m}{2}$ $\quad$ عرض رأس $=(\frac{m}{2})^2-m(\frac{m}{2})+1=1-\frac{m^2}{4}$

① $m=2 \to$ عرض رأس $=1-\frac{4}{4}=\boxed{-\frac{5}{4}}$ $\quad$ ② $m=-1 \to$ عرض رأس $=1-\frac{1}{4}=\boxed{\frac{3}{4}}$

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$y=ax^2+2x+a$ $\quad a=\frac{1\pm\sqrt{4+4a^2}}{2\times 8}=\frac{1\pm 2a}{16}$ $\quad a_1=2, a_2=-\frac{1}{8}$

min شود $\to a>0 \Rightarrow \boxed{a=2}$

$$\min\begin{cases}-\frac{b}{2a}=-\frac{2}{2a}\\ a(\frac{1}{4a^2})+2(-\frac{2}{2a})+a=\frac{1}{4a}-\frac{1}{2a}+a=\frac{-1+4a^2}{4a}=\frac{4a^2-1}{4a}=\frac{7}{8}\end{cases}\Rightarrow$$

$2\cancel{8}a=2\cancel{2}a^2-7\cancel{2} \Rightarrow 2\cancel{2}a^2-2\cancel{8}a-7\cancel{2}=0 \Rightarrow 8a^2-7a-18=0 \quad \Delta=49+4\times 7\times 9=49\times 5$

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$x^2-(a+1)x+a=0 \to \alpha+\beta=a+1, \alpha\times\beta=a \quad \alpha=2n-1, \beta=2n+1$

$(2n-1)+(2n+1)=a+1 \Rightarrow a=4n-1$ ✱ $\quad (2n-1)(2n+1)=a \Rightarrow 4n^2-1=a$ ✱✱

✱, ✱✱ $\to 4n\cancel{-2}=4n^2\cancel{-2} \Rightarrow 4n(n-1)=0 \xrightarrow{n\geq 1} n=1 \to a=4n-1 \Rightarrow a=3$ $\quad \alpha=1$ ($2-1$), $\beta=3$ ($2+1$)

$P_1=\alpha\times\beta=3$ $\quad x^2-(2a+1)x+b=0 \xrightarrow{a=2} x^2-10x+b=0 \to P+q=10, P\times q=b$

$P=2k, q=2k+2$ $\quad 2k+(2k+2)=10 \Rightarrow 4k+2=10 \Rightarrow k=2$ $\quad P=2(2)=4$ $\quad q=2(2)+2=6$

$P_2=Pq=24$ $\quad P_2-P_1=24-3=21$

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$y=-ax^2+ax+2$ $\quad \frac{a}{2a}=\frac{1}{2}$

$y=-a(\frac{1}{2})^2+a(\frac{1}{2})+2=\frac{a}{4}+2$

$y_{v_1}=2b(x_{v_1})^2-b(x_{v_1})-1 \Rightarrow$

$\frac{a}{4}+2=2b(\frac{1}{2})^2-b(\frac{1}{2})-1 \Rightarrow \frac{a}{4}+2=-1$

$\frac{a}{4}=-1-2 \Rightarrow \boxed{a=-12}$

$x_2=-\frac{-b}{4b}=\frac{1}{4}$

$y_2=2b(\frac{1}{4})^2-b(\frac{1}{4})-1 \Rightarrow y_2=-\frac{b}{8}-1=\overset{-12}{-a}(\frac{1}{4})^2+\overset{-12}{a}(\frac{1}{4})+2 \Rightarrow b=-6$

$b-a=(-6)-(-12)=6$

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$\alpha+\beta=-\frac{4}{2a} \Rightarrow \alpha(\alpha+\beta)=-\frac{4}{2a} \Rightarrow \alpha^2+\alpha\beta=-\frac{4}{2a}$ ✱

$\alpha\times\beta=\frac{\beta}{2a\alpha} \xrightarrow{\div\beta} \alpha=\frac{1}{2a\alpha} \Rightarrow$

$2a\alpha^2=1 \Rightarrow \alpha=\pm\frac{1}{a}$

$\alpha\beta=\frac{\beta}{2a\times\frac{1}{a}}=\frac{\beta}{a}$

① $\alpha^2+\alpha\beta=-\frac{4}{2a} \Rightarrow (\frac{1}{a})^2+\frac{\beta}{a}=-\frac{4}{2a} \Rightarrow \beta=-1, \alpha=\frac{1}{a}$ $\quad \beta<\alpha$ ✗

② $\alpha^2+\alpha\beta=-\frac{4}{2a} \Rightarrow (-\frac{1}{a})^2+(-\frac{\beta}{a})=-\frac{4}{2a} \Rightarrow \beta=1, \alpha=-\frac{1}{a}$ $\quad \beta>\alpha$ ✓

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$x_v=-\frac{b}{2a}=\frac{2}{a}$

$y_v=-a(\frac{2}{a})^2+4(\frac{2}{a})+1 \Rightarrow y_v=\frac{9}{a}$ $\quad$ نامساوی $\to (\frac{2}{a},\frac{9}{a})$

$a+b=-\frac{-(a^2+b^2-12)}{1} \Rightarrow a+b=a^2+b^2-12$ ① $\quad a\times b=\frac{a+b-1}{1} \xrightarrow{②} a\times b=a+b-1 \Rightarrow$

② $a+b=ab+1$ $\quad a^2+b^2=(a+b)^2-2ab \xrightarrow{①} a+b=[(a+b)^2-2ab]-12 \xrightarrow{③}$

$[ab+1]=[(ab+1)^2-2ab]-12$ $\quad$ ④ $S=P+1 \to P=S-1$ $\quad$ ① $S=(a^2+b^2)-12, a^2+b^2=S^2-2P$

$S=(S^2-2P)-12 \xrightarrow{P=S-1} S=S^2-2(S-1)-12 \Rightarrow S=S^2-2S-10=0 \Rightarrow (S-5)(S+2)=0$

$S=5$ یا $S=-2$ $\xrightarrow{\text{قبول}} S=a+b \geq 1+1=2$ $\quad S=-2$ ✗ $\quad S=5$ ✓

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