آزمون پودمانی

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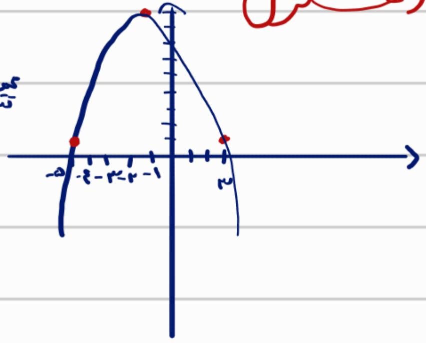
$(-1,9)$ ← رأس $\qquad \frac{-b}{2a}=-1 \to y=9 \to$ عرض رأس

$(3,1) \qquad 9\underset{\frac{1}{2}}{a}+3\underset{5}{b}+c=1 \to -18.5$

$9-1=8 \to 9+8=17$



$$\begin{array}{r} 25a-5b+c=1 \\ -\quad a-b+c=9 \\ \hline 24a-4b+0=-8 \end{array}$$

$\boxed{6a-b=-2} \rightsquigarrow \frac{-b}{2a}=\frac{-6a-2}{2a}=\frac{-3a-1}{a}=-1 \to -a=-3a-1$

$2a=1 \to \boxed{a=\frac{1}{2}}$

$6\underset{\frac{1}{2}}{a}+2=b \to \boxed{b=5} \qquad \boxed{c=-18.5} \rightsquigarrow \frac{1}{2}x^2+5x-18.5=y$

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$(x-\alpha)(x-\beta)$

$\alpha\beta=\frac{c}{a}=\frac{m+4}{2}>0 \to \begin{array}{c|c} \multicolumn{2}{c}{-4} \\ - & + \end{array} \rightsquigarrow m>-4$

$\alpha+\beta>0 \to \frac{-b}{a}>0 \to \frac{-m}{2}>0 \to \begin{array}{c|c} \multicolumn{2}{c}{0} \\ + & - \end{array} \rightsquigarrow m<0$

$\rightsquigarrow m<0$ و $m>-4$

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$\alpha+\beta=\frac{1}{\alpha\beta} \rightsquigarrow \frac{-b}{a}=\frac{a}{c} \to a^2=-bc \rightsquigarrow 9=-(2m+1)(2-m)$

$9=-4m+2m^2+2-m$

$0=2m^2-5m-7$

$m^2-5m-14 \rightsquigarrow (m-7)(m+2)$

$\boxed{\frac{7}{2}} \qquad \boxed{-1}=\frac{-2}{2}$

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$x_1=\alpha$ و $x_2=\beta \rightsquigarrow x^2-x-4$

$\Delta=b^2-4ac=1-4(1)(-4)=17$

دو ریشه $\frac{-b\pm\sqrt{\Delta}}{2a}$: $\quad \frac{1+\sqrt{17}}{2}=\alpha \qquad \frac{1-\sqrt{17}}{2}=\beta$

$\alpha^3+\frac{1}{\beta}=\frac{1+17\sqrt{17}}{8}+\frac{2}{1-\sqrt{17}}$

$\rightsquigarrow \beta^3+\frac{1}{\alpha}=\frac{1-17\sqrt{17}}{8}+\frac{2}{1+\sqrt{17}}$

$\frac{-2-34\sqrt{17}}{-16}+\frac{2+2\sqrt{17}}{-16}=\frac{-1-17\sqrt{17}+1+\sqrt{17}}{-8}=\frac{-16\sqrt{17}}{-8}=2\sqrt{17}$

$\frac{-1+17\sqrt{17}}{-8}+\frac{-2+2\sqrt{17}}{-8}=\frac{-3+19\sqrt{17}}{-8}$

$(x-2\sqrt{17})\left(x+\frac{3-19\sqrt{17}}{8}\right)$

$x^2+\frac{3-35\sqrt{17}}{8}x-6\sqrt{17}+464$

$$\left(\sqrt[3]{x}+\frac{1}{\sqrt[3]{x}}\right)^3-1 \qquad \frac{\sqrt[3]{x^4}+1+\sqrt[3]{x^2}}{\sqrt[3]{x^2}}=\left(\frac{\sqrt[3]{x^5}+\sqrt[3]{x^4}+x}{x}\right)\left(\sqrt[3]{x^3}-1\right)$$

$$\left(\frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}}\right)^3-1$$

$$\frac{\sqrt[3]{x^8}+x+x\sqrt[3]{x^4}}{x}+\frac{-\sqrt[3]{x^5}-\sqrt[3]{x}-x}{x}=\frac{x^2\sqrt[3]{x}+x\sqrt[3]{x^2}-x\sqrt[3]{x^2}-\sqrt[3]{x}}{x}$$

$$=\frac{x^2\sqrt[3]{x}-\sqrt[3]{x}}{x}\quad \frac{\sqrt[3]{x}(x^2-1)}{x}-\frac{2x\sqrt[3]{x}}{x}=\frac{\sqrt[3]{x}\,(x^2-1-2x)}{x}$$

x^2+1-2x یادش بود

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$\alpha=3\beta \lessgtr^{+2}_{-2} \quad \alpha\beta=\frac{c}{a} \qquad ax^2+\boxed{b}x+c=3x^2-\underline{a}x+4$

$\beta \qquad 3\beta^2=\frac{4}{3}\to\beta^2=\frac{4}{9}\to\beta=\pm\frac{2}{3}$

$\alpha+\beta=3\beta+\beta=4\beta=\frac{-b}{a}=\frac{+a}{3}\to 12\beta=+a$

$12\times\frac{2}{3}=8=-a\to a=+8 \to 3\times\frac{4}{9}-8\times\frac{2}{3}+4=\frac{4}{3}-\frac{16}{3}+\frac{12}{3}=0$ $=\frac{32}{3}$

$12\times-\frac{2}{3}=-8=-a\to a=8 \to 3\times\frac{4}{9}+8\times\left(-\frac{2}{3}\right)+4=0$

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$(x-\alpha)(x-\beta)$

$\alpha \quad \beta$

$8-(-8)=16$

$ax^2+(3+2a)x=y$

$ax^2+(3+2a)x\geq 0$

که به ازای x های مثبت

در کل نباید به ازای $x>0$ ، $y<0$ داشته باشیم

$a=0 \to 3x\geq 0\to$ که به ازای x های منفی جواب نمی‌دهد ✗

$a=-1\to -x^2+x\geq 0\to \underset{0}{x}\,\underset{1}{(-x+1)}$ ✗

$a=-\frac{3}{2}\to -\frac{3}{2}x^2\geq 0$ ✗

فقط به ازای a های منفی جواب نیست ← $a=-2\to -2x^2-x\geq 0\to x(-2x-1)$ ✗

$a=1\to x^2+5x\to \underset{0}{x}\,\underset{-5}{(x+5)}$ $\quad + \mid - \mid +$

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① محور تقارن $\Rightarrow \frac{-a}{2}$ $\qquad \frac{-a}{2}=-1\to\frac{a}{2}=1\to\boxed{a=2}$

② محور تقارن دو $\Rightarrow \frac{+2}{-2}=-1$ $\qquad -x^2-2x+b=1$

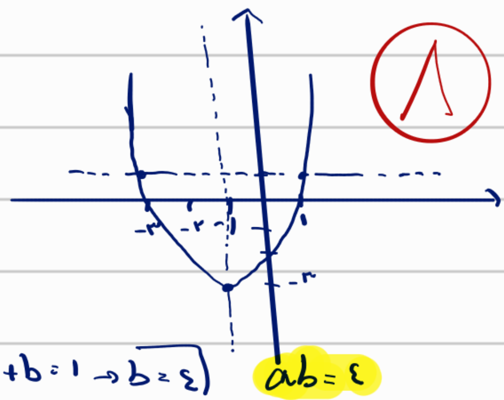
① رأس $\to y=x^2+2x-2\to$ رأس $\left|\begin{array}{l}x=-1\\ y=-3\end{array}\right.$ $\qquad -x^2-2x+b-1$

$\Delta>0$

$1=x^2+2x-3 \qquad 4-4\times(-1)(b-1)$

$0=x^2+2x-3 \qquad 4+4b-4\Rightarrow 4b>0$

$(x+3)(x-1)\to x=1,\ x=-3 \qquad -x^2-2x+b=1\to -9+6+b=1\to b=4$



$ab=4$

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$\alpha_1 + \frac{1}{2} = \alpha_2$

$\beta_1 + \frac{1}{2} = \beta_2$

$\alpha_1\beta_1 = \frac{c}{a} = \frac{b}{2}$ و $\alpha_1+\beta_1 = -\frac{b}{a} = \frac{a}{2}$

$\alpha_2\beta_2 = \frac{c}{a} = (\alpha_1+\frac{1}{2})(\beta_1+\frac{1}{2}) = -3$

$$\underbrace{\alpha_1\beta_1}_{\frac{b}{2}} + \underbrace{\frac{1}{2}\alpha_1 + \frac{1}{2}\beta_1}_{\frac{1}{2}(\alpha_1+\beta_1)} + \frac{1}{4} = -3 \to \frac{2b+a}{4} = \frac{-12}{4} \to 2b + \underbrace{a}_{-3} = -13 \to \boxed{b = 5}$$

$$\alpha_2+\beta_2 = \alpha_1 + \frac{1}{2} + \beta_1 + \frac{1}{2} = \underbrace{\alpha_1+\beta_1}_{\frac{a}{2}} + 1 = \frac{-a}{2a} \to \frac{a}{2} + 1 = -\frac{1}{2} \to \boxed{a = -3}$$

$$\left[\frac{ab}{4}\right] = \frac{-15}{4} = -3\frac{3}{4} \to -4$$

α و β_1, β_2

$\alpha\beta_1 = m \rightsquigarrow (-6-\beta_1)(\beta_1) = -\beta_1^2 - 6\beta_1 = m \to -\beta_1(+\beta_1+6) = m$

$\alpha\beta_2 = -3m \rightsquigarrow (-6-\beta_1)(\beta_1+4) = -6\beta_1 - 24 - \beta_1^2 - 4\beta_1 \to -\beta_1^2 - 10\beta_1 - 24 = -3m$

$\boxed{\alpha = -5} \leftarrow \alpha + \underbrace{\beta_1}_{-1} = -6 \to \alpha = -6 - \beta_1 \to -2 - \beta_2 = -6 - \beta_1 \to \underbrace{\beta_2}_{\beta_1+4} - \beta_1 = 4$

$\beta_1^2 + 10\beta_1 + 24 = 3m$

$(\beta_1+4)(\beta_1+6) = 3m$

$\alpha + \beta_2 = -2 \rightsquigarrow -6 - \beta_1 + \beta_2 = -2 \to \beta_2 - \underbrace{\beta_1}_{-1} = 4 \rightsquigarrow \boxed{\beta_2 = 3}$

$\beta_2 - \beta_1 = 4$

$\beta_2 = \beta_1 + 4 \to$ ادامه با صورتی

$\frac{-\beta_1 \times 3m}{\beta_1+4} = m$

$\frac{-3\beta_1}{\beta_1+4} = 1 \rightsquigarrow \beta_1 + 4 = -3\beta_1$

$4\beta_1 = -4 \to \boxed{\beta_1 = -1}$

ادامه با قرمز

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