

$$\frac{-b}{2a} = -1 \Rightarrow b = 2a \Rightarrow x = -1 \Rightarrow a - b + c = 9 \Rightarrow -a + c = 9 \Rightarrow \begin{cases} x = 3 \Rightarrow 9a + 3b + c = 1 \\ 19a = -1 \Rightarrow a = -\frac{1}{19} \Rightarrow b = 2(-\frac{1}{19}) = -\frac{2}{19} \\ c = 9 - 1 = \frac{16}{19} \end{cases}$$

$$\boxed{y = -\frac{1}{19}x^2 - \frac{2}{19}x + \frac{16}{19}}$$

$$y = 2x^2 + m x + m + 1 = 0 \Rightarrow \text{دو ریشه دارد} \Rightarrow m^2 - 4(2m+1) \geq 0 \Rightarrow m^2 - 8m - 4 \geq 0 \Rightarrow (m+4)(m-12) \geq 0 \Rightarrow \frac{-4}{+} \mid \frac{12}{+}$$

① $m \in (-\infty, -4) \cup (12, +\infty) \Rightarrow \begin{cases} \textcircled{1} S = \frac{-m}{2} \Rightarrow m < 0 \Rightarrow \textcircled{2} \frac{m+1}{2} \Rightarrow m > -1 \end{cases} \Rightarrow m \in (-1, 0) \Rightarrow \textcircled{1}, \textcircled{2}, \textcircled{3}$ اشک = $-1 < m < 0$

$$2x^2 + (2m-1)x + 2-m = 0 \Rightarrow \text{دو ریشه دارد} \Rightarrow (2m-1)^2 - 4(2-m) \geq 0 \Rightarrow 4m^2 - 4m - 1 \geq 0 \Rightarrow m = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow \frac{-1 \pm \sqrt{5}}{2} \mid \frac{-1 \pm \sqrt{5}}{2} \Rightarrow m \in (-\infty, -1 - \frac{\sqrt{5}}{2}) \cup (-1 + \frac{\sqrt{5}}{2}, +\infty)$$

② $\alpha + \beta = \frac{1}{\alpha\beta} \Rightarrow \frac{2m+1}{2} = \frac{1}{\frac{2-m}{2}} \Rightarrow 2m^2 - 4m - 1 = 0 \Rightarrow (2m-1)(m+1) = 0 \Rightarrow m = \frac{1}{2} \text{ یا } -1$

$$x^2 - x - 1 = 0 \Rightarrow S = 1, P = -1 \Rightarrow \begin{cases} x_1^3 + x_2^3 + \frac{1}{x_1} + \frac{1}{x_2} = (x_1 + x_2)^3 - 3(x_1 + x_2) + \frac{x_1 + x_2}{x_1 x_2} = 1 + 1 + \frac{-1}{-1} = 2 \end{cases}$$

$$(x_1^3 + \frac{1}{x_1})(x_2^3 + \frac{1}{x_2}) = (x_1 + x_2)^3 + x_1^2 + x_2^2 + \frac{1}{x_1 x_2} = -1 + 9 - \frac{1}{-1} = 9 \Rightarrow \boxed{y = x^2 - \frac{1}{9}x - \frac{22}{9}}$$

$$\sqrt{x} = t \Rightarrow (t + \frac{1}{t} + 1)(t-1) = t^2 - \frac{1}{t} \Rightarrow \sqrt{x} - \frac{1}{\sqrt{x}} = 2\sqrt{x} \Rightarrow \frac{\sqrt{x}-1}{\sqrt{x}} = 2\sqrt{x} \Rightarrow x-1 = 2x \Rightarrow x-2x-1 = 0 \Rightarrow \boxed{S = 2}$$

$$3x^2 - \alpha x + 1 = 0, \alpha = 2\beta \Rightarrow P = \frac{1}{\alpha} \Rightarrow 3\beta^2 = \frac{1}{\alpha} \Rightarrow \beta^2 = \frac{1}{9\alpha} \Rightarrow \beta = \frac{1}{3\sqrt{\alpha}} \Rightarrow \alpha = \frac{1}{9\beta^2} \Rightarrow \begin{cases} \textcircled{1} \frac{1}{\beta} = \frac{\alpha}{\beta} \Rightarrow \alpha = 1 \\ \textcircled{2} \frac{-1}{\beta} = \frac{\alpha}{\beta} \Rightarrow \alpha = -1 \end{cases} \Rightarrow 1 - (-1) = \textcircled{2}$$

$$y = ax^2 + (2+2a)x = \textcircled{1} a \Rightarrow \begin{cases} \textcircled{1} \text{ نمودار } y = ax^2 + (2+2a)x \text{ دارای یک ریشه مضرب است.} \\ \textcircled{2} a \neq \frac{1}{4} \end{cases}$$

③ $S \gg 0 \Rightarrow \frac{2+2a}{a} > 0 \Rightarrow \frac{2}{a} > 0 \Rightarrow a > 0$ اشک = $\boxed{Q}$ (هیچ مقدار)

$$y = -x^2 - 2x + b, y = x^2 + ax - 2 \Rightarrow \frac{-b}{2a} = \frac{2}{-2} = -1 \Rightarrow a = 2 \Rightarrow \begin{cases} |1 - \frac{-x^2 - 2x + b}{x^2 + ax - 2}| = 1 \Rightarrow 1 = \frac{-x^2 - 2x + b}{x^2 + ax - 2} \Rightarrow b = 1 \end{cases} \Rightarrow ab = 2 \times 1 = \textcircled{1}$$

$$\begin{aligned}
 & \{ ax^2 + ax - 4 = 0 \Rightarrow S = \frac{-a}{1a} = -1, P = \frac{-4}{1} = -4 \} \quad \{ x^2 - ax + b = 0 \Rightarrow S = \alpha + \frac{1}{\beta} + \beta + \frac{1}{\alpha} = \frac{-1}{1} + \frac{1}{1} + \frac{1}{1} = \frac{-(-a)}{1} \Rightarrow a = 1 \quad -9 \\
 & P = \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\beta}(\alpha + \beta) + \frac{1}{\alpha} = \frac{-4}{1} \Rightarrow -4 + \frac{1}{1}\left(\frac{-1}{1}\right) + \frac{1}{1} = \frac{-4}{1} \Rightarrow b = -1 \quad \{ ab = (-1)(1) = -1 \Rightarrow \sqrt{\frac{ab}{1}} = \sqrt{\frac{-1}{1}} = \sqrt{-1} = \textcircled{-1}
 \end{aligned}$$

$$\begin{aligned}
 & x^2 + 4x + m = 0 \Rightarrow S = -4 = \alpha + \beta \quad \{ x^2 + 4x - 3m = 0 \Rightarrow S = -4 \Rightarrow \alpha = -4 - \beta \quad \{ \textcircled{1}, \textcircled{2} \Rightarrow -4 - \beta = -4 - 8 \Rightarrow \boxed{8 - \beta = 4}
 \end{aligned}$$