

$$\frac{-b}{2a} = -1 \Rightarrow b = 2a \Rightarrow x_2 = -1 \Rightarrow a - b + c = 9 \Rightarrow -a + c = 9 \Rightarrow \begin{cases} x_2 = -1 \\ x_1 = 3 \end{cases} \Rightarrow 9a + 3b + c = 1 \Rightarrow 19a = -1 \Rightarrow a = -\frac{1}{19} \Rightarrow b = 2(-\frac{1}{19}) = -\frac{2}{19}$$

$$c = 9 - 1 = \frac{16}{19}$$

$$y = \frac{1}{19}x^2 - \frac{2}{19}x + \frac{16}{19}$$

$$y = 2x^2 + m x + n = 0 \Rightarrow \text{دو ریشه دارد} \Rightarrow m^2 - 4(2n) \geq 0 \Rightarrow m^2 - 8n \geq 0 \Rightarrow (m+4)(m-12) \geq 0 \Rightarrow \frac{-4}{+} \mid \frac{12}{-}$$

① $m \in (-\infty, -4) \cup (12, +\infty)$ $\Rightarrow \begin{cases} 1) S = \frac{-m}{2} > 0 \Rightarrow m < 0 \\ 2) \frac{m+4}{2} > 0 \Rightarrow m > -4 \end{cases} \Rightarrow m \in (-4, 0) \Rightarrow \text{اشتراک 1 و 2}$

$$2x^2 + (2m-1)x - m = 0 \Rightarrow \text{دو ریشه دارد} \Rightarrow (2m-1)^2 - 4(2)(-m) \geq 0 \Rightarrow 4m^2 - 4m + 1 + 8m \geq 0 \Rightarrow 4m^2 + 4m + 1 \geq 0 \Rightarrow (2m+1)^2 \geq 0$$

$$\frac{-1 \pm \sqrt{1+8m}}{4} = \frac{-1 \pm \sqrt{1+8m}}{4} \Rightarrow \frac{-1 \pm \sqrt{1+8m}}{4} \geq 0 \Rightarrow \frac{-1 \pm \sqrt{1+8m}}{4} \geq 0 \Rightarrow m \in (-\infty, -\frac{1}{8}) \cup (-\frac{1}{8}, +\infty)$$

② $\alpha + \beta = \frac{1}{\alpha\beta} \Rightarrow \frac{2m-1}{-m} = \frac{1}{\frac{-m}{2}} \Rightarrow 2m-1 = -2 \Rightarrow m = -\frac{1}{2}$

$$x_1^2 - x_2^2 = 0 \Rightarrow S = 1, P = -\frac{1}{2} \Rightarrow \begin{cases} x_1^3 + x_2^3 + \frac{1}{x_1} + \frac{1}{x_2} = (x_1 + x_2)^3 - 3(x_1 + x_2) + \frac{1}{x_1} + \frac{1}{x_2} = 1 + 1 + \frac{-1}{-1} = \frac{2}{-1} = -2 = S \end{cases}$$

$$(x_1^3 + \frac{1}{x_1})(x_2^3 + \frac{1}{x_2}) = (x_1 x_2)^3 + x_1^2 + x_2^2 + \frac{1}{x_1 x_2} = -\frac{1}{2} + 1 + \frac{1}{-\frac{1}{2}} = -\frac{1}{2} + 1 - 2 = -\frac{3}{2}$$

$$\sqrt{x} = t \Rightarrow (t + \frac{1}{t} + 1)(t-1) = t^2 - \frac{1}{t} \Rightarrow \sqrt{x} - \frac{1}{\sqrt{x}} - 2\sqrt{x} = \frac{\sqrt{x}-1}{\sqrt{x}} = 2\sqrt{x} \Rightarrow x-1 = 2x \Rightarrow x-2x-1 = 0 \Rightarrow \sqrt{S} = 2$$

$$3x^2 - \alpha x + c = 0, \alpha = 2\beta \Rightarrow P = \frac{c}{\alpha} = \frac{c}{2\beta} = \frac{c}{\beta} \Rightarrow \beta = \frac{c}{\alpha} \Rightarrow \alpha = \frac{2c}{\beta} \Rightarrow \alpha = \frac{2c}{\frac{c}{\alpha}} \Rightarrow \alpha = 2\alpha \Rightarrow \alpha = 0$$

$$y = a x^2 + (2+2a)x = 0 \Rightarrow \begin{cases} 1) \Delta \geq 0 \Rightarrow 4 + 4a^2 + 12a \geq 0 \Rightarrow (2a+3)^2 \geq 0 \Rightarrow a \neq -\frac{3}{2} \end{cases}$$

② $S \geq 0 \Rightarrow \frac{-(2+2a)}{a} \geq 0 \Rightarrow \frac{-2(1+a)}{a} \geq 0 \Rightarrow \frac{1+a}{a} \leq 0 \Rightarrow \frac{1}{a} \leq -1 \Rightarrow a \leq -1$

$$y = -x^2 - 2x + b, y = x^2 + \alpha x - 2 \Rightarrow \frac{-b}{2a} = \frac{2}{-2} = -1 \Rightarrow a = 2 \Rightarrow \begin{cases} 1) -x^2 - 2x + b = x^2 + \alpha x - 2 \Rightarrow b = \alpha^2 + 4 \end{cases} \Rightarrow \alpha = 2$$

$$\begin{aligned}
 & \{ ax^2 + ax - 4 = 0 \Rightarrow S = \frac{-a}{1a} = -\frac{1}{1}, P = \frac{-4}{1} = -4 \} \quad \{ x^2 - ax + b = 0 \Rightarrow S = \alpha + \frac{1}{\beta} + \beta + \frac{1}{\alpha} = \frac{-1}{1} + \frac{1}{1} + \frac{1}{1} = \frac{-(-a)}{1} = a = 1 \quad -9 \\
 & P = \left(\alpha + \frac{1}{\beta}\right) \left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\beta}(\alpha + \beta) + \frac{1}{\alpha} = \frac{-4}{1} \Rightarrow -4 + \frac{1}{1} \left(\frac{-1}{1}\right) + \frac{1}{1} = \frac{-4}{1} \Rightarrow b = -1 \quad \{ ab = (-1)(1) = -1 \Rightarrow \sqrt{\frac{ab}{1}} = \sqrt{\frac{-1}{1}} = \sqrt{-1} = \textcircled{-1}
 \end{aligned}$$

$$\begin{aligned}
 & x^2 + 4x + m = 0 \Rightarrow S = -4 = \alpha + \beta \quad \{ x^2 + 4x - 3m = 0 \Rightarrow S = -4 \Rightarrow \alpha = -4 - \beta \quad \{ \textcircled{1}, \textcircled{2} \Rightarrow -4 - \beta = -4 - 8 = -12 \Rightarrow \boxed{\gamma - \beta = 12} \quad -10
 \end{aligned}$$