۱)

$\frac{-b}{2a} = -1 \Rightarrow b = 2a \;/\; x=-1 \Rightarrow a - b + c = 9 \Rightarrow -a + c = 9$

$x = 3 \Rightarrow 9a + \underbrace{3b}_{6a} + \underbrace{c}_{9+a} = 1 \Rightarrow 16a = -8 \Rightarrow a = \frac{-1}{2} \Rightarrow b = 2\left(\frac{-1}{2}\right) = -1$

$c = 9 - \frac{1}{2} = \frac{17}{2}$

$$\boxed{y = \frac{-1}{2}x^2 - x + \frac{17}{2}}$$

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۲)

$y = 2x^2 + mx + m + 6 = 0 \Rightarrow$ دو ریشه دارد $\Rightarrow m^2 - 4(2m + 12) > 0$

$\Rightarrow m^2 - 8m - 48 > 0 \Rightarrow (m+4)(m-12) > 0 \Rightarrow$

	-4		12	
$+$		$-$		$+$

$\Rightarrow$ ① $m \in (-\infty, -4) \cup (12, +\infty)$

② $S = -\frac{m}{2} > 0 \Rightarrow m < 0$

③ $\frac{m+6}{2} > 0 \Rightarrow m > -6$

اشتراک ① و ② و ③ $\Rightarrow -6 < m < -4$

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۳)

$3x^2 + (2m-1)x + 2 - m = 0 \Rightarrow$ دو ریشه دارد $\Rightarrow 4m^2 + 1 - 4m - 4(6 - 3m) > 0$

$\Rightarrow 4m^2 + 8m - 23 > 0 \Rightarrow m = \frac{-8 \pm \sqrt{64 - 4(-92)}}{8} = \frac{-8 \pm \sqrt{2^4(2^2 + 23)}}{8}$

$= \frac{-2 \pm \sqrt{27}}{2} \Rightarrow$

	$-1 - \frac{\sqrt{27}}{2}$		$-1 + \frac{\sqrt{27}}{2}$	
$+$		$-$		$+$

$\Rightarrow$

① $m \in (-\infty, \underbrace{-1 - \frac{\sqrt{27}}{2}}_{\simeq -\frac{7}{2}}) \cup (\underbrace{-1 + \frac{\sqrt{27}}{2}}_{\simeq \frac{5}{2}}, +\infty)$

② $\alpha + \beta = \frac{1}{\alpha\beta} \Rightarrow \frac{-2m+1}{3} = \frac{1}{\frac{2-m}{3}} \Rightarrow 2m^2 - 5m - 7 = 0 \Rightarrow (2m - 7)(m + 1) = 0$

$\Rightarrow m \begin{cases} ② \; -1 \; \times \\ ② \; \frac{7}{2} \end{cases}$ اشتراک ① و ② $\Rightarrow m = \frac{7}{2}$

(4

$x^2 - x - 4 = 0 \Rightarrow S=1,\ P=-4$

$x_2^3 + x_1^3 + \frac{1}{x_2} + \frac{1}{x_1} = (\underbrace{x_1+x_2}_{1})^3 - 3(-4)(\underbrace{x_1+x_2}_{1}) + \frac{\overset{1}{x_1+x_2}}{\underset{-4}{x_1x_2}} =$

$1+12+\frac{-1}{4} = \frac{51}{4} = S$

$(x_2^3+\frac{1}{x_1})(x_1^3+\frac{1}{x_2}) = (\overset{-4}{x_1x_2})^3 + \underbrace{x_1^2+x_2^2}_{1-2(-4)=9} + \frac{1}{\underset{-4}{x_1x_2}} = -64+9-\frac{1}{4} = -\frac{221}{4}$

$y = x^2 - \frac{51}{4}x - \frac{221}{4}$

(5

$\sqrt[3]{x^2} = t \Rightarrow (t+\frac{1}{t}+1)(t-1) = t^2 - \frac{1}{t} \Rightarrow \sqrt[3]{x^4} - \frac{1}{\sqrt[3]{x^2}} = 2\sqrt[3]{x}$

$\Rightarrow \frac{\sqrt[3]{x^6-1}}{\sqrt[3]{x^2}} = 2\sqrt[3]{x} \Rightarrow x^2-1=2x \Rightarrow x^2-2x-1=0 \Rightarrow S=2$

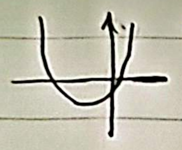
(6

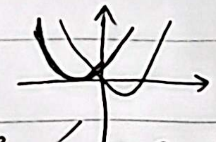
$3x^2 - \alpha x + 4 = 0,\ \alpha = 3\beta \Rightarrow P = \frac{4}{3} \Rightarrow 3\beta^2 = \frac{4}{3} \Rightarrow \beta^2 = \frac{4}{9} \Rightarrow \beta \begin{cases} ①\ \frac{2}{3} \\ ②\ -\frac{2}{3} \end{cases}$

$\Rightarrow \alpha \begin{cases} ①\ 2 \\ ②\ -2 \end{cases} \Rightarrow S \begin{cases} ①\ \frac{8}{3} = \frac{\alpha}{3} \Rightarrow \alpha = 8 \\ ②\ -\frac{8}{3} = \frac{\alpha}{3} \Rightarrow \alpha = -8 \end{cases} \Rightarrow 8-(-8)=16$

(7

$y = ax^2 + (3+2a)x \Rightarrow ①\ a>0$

② 



$\Delta \geq 0 \Rightarrow 9 + 4a^2 + 12a \geq 0$

$\Rightarrow (2a+3)^2 \geq 0 \Rightarrow a \neq -\frac{3}{2}$ ③

④ $S \geq 0 \Rightarrow \frac{-3-2a}{a} \geq 0 \Rightarrow -\frac{3}{2} > a$

①، ②، ③ و ④ اشتراک $\Rightarrow \varnothing$ هیچ مقدار

DATE:

$y=-x^2-2x+b$, $y=x^2+ax-2 \Rightarrow \frac{-b}{2a}=\frac{2}{-2}=\frac{-a}{2} \Rightarrow a=2$ (1

$\begin{cases} 1=-x^2-2x+b \\ 1=x^2+2x-2 \end{cases} \Rightarrow 2=-2+b \Rightarrow b=4 \qquad ab=2\times 4=8$

$2ax^2+ax-4=0 \Rightarrow S=\frac{-a}{2a}=\frac{-1}{2}$, $P=\frac{-4}{2}=-2$ (9

$2x^2-ax+b=0 \Rightarrow S=\alpha+\frac{1}{2}+\beta+\frac{1}{2}=-\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

$=\frac{-(-a)}{2} \Rightarrow a=1$

$P=(\alpha+\frac{1}{2})(\beta+\frac{1}{2})=\alpha\beta+\frac{1}{2}(\alpha+\beta)+\frac{1}{4}=\frac{+b}{2}$

$\Rightarrow -2+\frac{1}{2}(-\frac{1}{2})+\frac{1}{4}=\frac{+b}{2} \Rightarrow b=-4$

$ab=(-4)\times 1=-4 \Rightarrow \left[\frac{\overbrace{ab}^{-4}}{4}\right]=\left[-\frac{4}{2}\right]=-2$

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1) $x^2+4x+m=0 \Rightarrow S=-4 \Rightarrow \alpha=-4-\beta$ (10
ریشه مشترک

2) $x^2+2x-3m=0 \Rightarrow S=-2 \Rightarrow \alpha=-2-\delta$

$(1,\delta) \Rightarrow -4-\beta=-2-\delta \Rightarrow \delta-\beta=4$