Date: ....

یکتا انوری - دهم B

۱-

$y=ax^2+bx+c \qquad A=(-1,9) \qquad B=(3,1)$

$\frac{-b}{2a}=-1 \to 2a=b \; *$

$\xrightarrow{x=-1} a-b+c=9 \xrightarrow{*} c-a=9 \to c=a+9$

$\xrightarrow{x=3} 9a+3b+c=1 \Rightarrow 9a+6a+a=-8 \Rightarrow 16a=-8 \to a=-\frac{1}{2}$ ↗ $b=-1$ ↘ $c=\frac{17}{2}$

$y=-\frac{1}{2}x^2-x+\frac{17}{2}$

۲-

+ ۲ ریشه: ① $a>0$ ② $\Delta>0$ ③ $S>0$ ④ $P>0$

$y=2x^2+mx+m+6=0 \to m^2-8(m+6)>0 \to m^2-8m-48>0$

$\to (m-12)(m+4)>0 \to m\in(-\infty,-4)\cup(12,+\infty)$

$-\infty$		-4		12		$+\infty$
	$+$	0	$-$	0	$+$	

$\frac{-m}{2}>0 \to m<0$

$\frac{m+6}{2}>0 \to m+6>0 \to m>-6$

اشتراک $-6<m<-4 \Rightarrow m\in(-6,-4)$

۳-

$\Delta>0 - (2m-1)^2-12(2-m)>0 \to 4m^2-4m+1-24+12m>0$

$4m^2+8m-23>0 \xrightarrow{x_1,x_2} \frac{-8\pm\sqrt{64+16\times 23}}{8}=\frac{-2\pm\sqrt{27}}{2}$

	$\frac{-2-\sqrt{27}}{2}$		$\frac{-2+\sqrt{27}}{2}$	
$+$		$-$		$+$

$m>\frac{-2+\sqrt{27}}{2} \approx \frac{3}{2}$, $m<\frac{-2-\sqrt{27}}{2} \approx -\frac{7}{2}$ $\boxed{*}$

$\alpha+\beta=\frac{1}{\alpha\cdot\beta} \to \frac{-2m}{3}=\frac{3}{2-m} \to 2-4m-m+2m^2=9 \to 2m^2-5m-7=0$

$(2m-7)(m+1)=0 \to m=-1 \,/\, \frac{7}{2} \xrightarrow{\boxed{*}} m=\frac{7}{2}$

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$$x^2 - x - 4 = 0 \longrightarrow S = x_1 + x_2 = 1$$
$$P = x_1 \cdot x_2 = -4$$

$$S'_2 = x_2^3 + x_1^3 + \frac{1}{x_1} + \frac{1}{x_2} = S^3 - 3SP + \frac{S}{P} = 1 - 3\times(-4)\times 1 - \frac{1}{4} =$$
$$13 - \frac{1}{4} = \frac{51}{4}$$

$$P'_2 = \left(x_2^3 + \frac{1}{x_1}\right)\left(x_1^3 - \frac{1}{x_2}\right) = (x_2 x_1)^3 + x_2^2 + x_1^2 + \frac{1}{x_1 \cdot x_2} =$$

$$= -64 + 1 + 8 - \frac{1}{4} = -55 - \frac{1}{4} = \frac{-221}{4} \qquad x^2 - Sx + P = x^2 - \frac{51}{4}x - \frac{221}{4}$$

۵-

$$\left(\frac{\sqrt[3]{x^4} + 1 + \sqrt[3]{x^2}}{\sqrt[3]{x^2}}\right)\left(\sqrt[3]{x^2} - 1\right) = 2\sqrt[3]{x} \longrightarrow \frac{x^2 - 1}{\sqrt[3]{x^2}} = 2\sqrt[3]{x}$$

$$x^2 - 2x - 1 = 0 \longrightarrow x_1 + x_2 = 2$$

۶-

$$3x^2 - \alpha x + 4 = 0 \ , \ \alpha \cdot 3\alpha = \frac{4}{3} \longrightarrow 3\alpha^2 = \frac{4}{3} \longrightarrow \alpha^2 = \frac{4}{9}$$

$$\alpha = -\frac{2}{3} \qquad \alpha = \frac{2}{3}$$

$$S_2 = -2 - \frac{2}{3} = \frac{-8}{3} \longleftarrow 3\alpha = -2$$
$\alpha = -8$

$$3\alpha = 2 \rightarrow S_1 = 2 + \frac{2}{3} = \frac{8}{3}$$
$\alpha = 8$

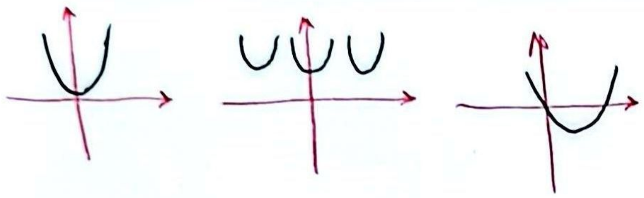
$$\longrightarrow 8 - (-8) = +16$$

Sibo

-7

$y = ax^2 + (3+2a)x$

هیچ مقدار (جواب):



$3 + 2a = 0$

$\text{I} \; a = -\frac{3}{2}$
$\text{II} \; a \geq 0$ } ✗

$\Delta < 0$
$a \geq 0$ $\Big| \to (2a+3)^2 < 0$ ✗

$\Delta \geq 0$
$a \geq 0$
$S = -\frac{(3+2a)}{a} \geq 0$

$-\frac{3}{2} \quad 0$
$- \quad + \quad -$ $\to a \in [-\frac{3}{2}, 0)$ ✗

-8

$y = -x^2 - 2x + b \qquad y = x^2 + ax - 2 \to y = x^2 + 2x - 2$

$\frac{2}{2} = \frac{a}{2} \to a = 2$ ✓

$1 = x^2 + 2x - 2 \to x^2 + 2x - 3 = 0 \to \underset{x=-3}{(x+3)}\underset{x=1}{(x-1)} = 0$

$\xrightarrow{x=1} -1 - 2 + b = 1 \to b = 4$

$\xrightarrow{x=-3} -9 + 6 + b = 1 \to b = 4$

$ab = 2 \times 4 = 8$

-9

$2x^2 - ax + b = 0$

$\alpha + \frac{1}{2} \,,\, \beta + \frac{1}{2}$

$S = \alpha + \beta + 1 = \frac{1}{2} \to \frac{a}{2} \to a = 1$

$P = (\alpha + \frac{1}{2})(\beta + \frac{1}{2}) = \frac{1}{4} + \frac{1}{2}(\alpha + \beta) + \alpha\beta = \alpha\beta = -3 = \frac{b}{2} \to \underline{b = -6}$

$2ax^2 + ax - 6 = 0$

ریشه ها: α, β $\qquad S = \alpha + \beta = \frac{-a}{2a} = \frac{-1}{2} \qquad P = \alpha\beta = \frac{-6}{\cancel{2a}_{\,2}} = -3$

$a = 1$
$b = -6$ $\quad \left[\frac{ab}{4}\right] = \left[\frac{-6}{4}\right] = \left[\frac{-3}{2}\right] = \underline{-2}$

-10

$x^2 + 6x + m = 0$

ریشه ها: α, β

$\alpha + \beta = -6$

$\alpha = -6, \beta$

$x^2 + 2x - 3\alpha = 0$

ریشه ها: α, m

$S = \alpha + m = \frac{-a}{2a} = \frac{-1}{2}$

$P = \alpha m = \frac{-6}{2 \cdot \cancel{2a}} = -3$

$\Rightarrow -2 - m = -6 - \beta \to m - \beta = 6 - 2 = \underline{4}$