نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

۱۷٫۵

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$x_S = -\frac{b}{2a} = -1 \Rightarrow b = 2a$ ✱ $\quad y_S = 9 = a(-1)^2 + (-1)b + c \Rightarrow c = 9 + a$

(۳و۱) $\quad 1 = a(3)^2 + b(3) + c = 9a + 3(2a) + (9+a) = 16a = -8 \rightarrow a = -\frac{1}{2}$

✱ $\quad b = 2a = -1 \quad , \quad c = 9 + a = \frac{17}{2} \Rightarrow y = -\frac{1}{2}x^2 - x + \frac{17}{2}$ ✓ ج

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$\Delta > 0 \rightarrow m^2 - 4(m+4) > 0 \rightarrow m^2 - 4m - 16 > 0 \Rightarrow (m+2)(m-12)$

$-2 \quad 12$

$S = \frac{-b}{a} = \frac{-m}{2} > 0 \rightarrow m < 0$

$P = \frac{c}{a} = \frac{m+4}{2} > 0 \rightarrow m > -4$

ج

$\rightarrow -4 < m < -2$ ✓

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$S = \frac{1-2m}{3} \quad , P = \frac{2-m}{3} \Rightarrow S = \frac{1}{P} \Rightarrow \frac{1-2m}{3} = \frac{3}{2-m}$

$(1-2m)(2-m) = 9 \Rightarrow 2m^2 - 5m + 2 = 9 \rightarrow 2m^2 - 5m - 7 = 0$

$\Delta = (-5)^2 - 4(2)(-7) = 81 \rightarrow m = \frac{5 \mp 9}{4} \begin{cases} m = \frac{7}{2} \checkmark \\ m = -1 \times \end{cases}$ (۱٫۵)

$m = -1 \rightarrow y = 3x^2 + 5x + 2 \rightarrow \Delta < 0$ غ ق ق

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$x^2 - x - 2 = 0 \Rightarrow S = 1 \quad , P = -2 \Rightarrow y_1 = x_2^3 + \frac{1}{x_1} \quad , y_2 = x_1^3 + \frac{1}{x_2}$

$\rightarrow P_{y_1, y_2} = \frac{(x_2^3 + 1)(x_1^3 + 1)}{P} = \frac{x_1^3 x_2^3 + x_1^3 + x_2^3 + 1}{4} = \frac{(x_1 x_2)^3 + 4S^3 - 3PS + 1}{4}$

$= \frac{-8 + 12 + 1}{-2} = \frac{-10}{2} = \frac{5}{1}$ ، $S_{y_1, y_2} = \frac{+5}{2} \Rightarrow y^2 + \frac{5}{2}y + \frac{5}{2} = 0$

(۱٫۵)

$\rightarrow 2y^2 + 5y + 5 = 0$

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$t = \sqrt[3]{x} \Rightarrow (t^2 + \frac{1}{t^2} + 1)(t^2 - 1) = 2t \Rightarrow (t^2 - 1)(t^2 + t + 1) = t^4 - 1$

$\rightarrow t^4 - 1 = 2t^2 \rightarrow t^4 - 2t^2 - 1 = 0 \xrightarrow{y = t^2} y^2 - 2y - 1 = 0 \rightarrow \Delta = 12 = 4\sqrt{3}$

$S = 2$ ✓

ج ✓

r و $3r$ $\Rightarrow$ (P) $3r^2 = \frac{4}{3} \to r^2 = \frac{4}{9} \Rightarrow r = \mp\frac{2}{3}$ $S \Rightarrow 4r = \frac{a}{3}$ → $a = \frac{2}{3} \to 4\times\frac{2}{3} = \frac{8}{3} \to a = 8$ → $a = -\frac{2}{3} \to a = -8$ $\to 8-(-8) = 16$ (۲)	۶
$x(ax+3+2a)=0 \xrightarrow{x=0} x = \frac{-3-2a}{a}$ $\xrightarrow{\text{ریشه دوم منفی نباشد}}$ $\frac{-3-2a}{a} \geq 0$ $\to -\frac{3}{2} \leq a < 0$ (۱)	۷
$x_{S_1} = -\frac{a}{2}$ $\quad x_{S_2} = -1 \to -\frac{a}{2} = -1 \to a = 2$ $\xrightarrow{y=1}$ $1 = x^2+2x-2 \to x^2+2x-3=0$ (دو ریشه داره ✓ $\Delta>0$) ، $1 = -x^2-2x+b \to x^2+2x+(1-b)=0$ $\to \Delta = 4-4(1-b) > 0 \to 4b>0 \to b>0 \Rightarrow$ دو نقطه با عرض ۱ مساوی $= 3 \to ab = 2\times3 = 6$ (۱٫۵)	۸
$2ax^2+ax-6=0 \to S = \frac{-a}{2a} = -\frac{1}{2}$ و $P = \frac{-6}{2a} = -\frac{3}{a}$ $2x^2-ax+b=0 \to S = \frac{a}{2}$ $\xrightarrow{r=\text{ریشه}}$ $r_1+\frac{1}{2}$ و $r_2+\frac{1}{2}$ $\quad (r_1+\frac{1}{2})(r_2+\frac{1}{2})$ (۱٫۷۵) $\to r_1+r_2+1 = \frac{a}{2}$ ($-\frac{1}{2}$) $\to a=1$ $\quad \to (r_1+\frac{1}{2})(r_2+\frac{1}{2}) = P + \frac{1}{2}S + \frac{1}{4} =$ ($-\frac{3}{a}$, $-\frac{1}{2}$) $\to \frac{b}{2} = -3 \to b = -6$ $\quad \to \frac{ab}{4} = \frac{1\times-6}{4} = -\frac{3}{2}$ سوال $[\frac{ab}{4}]$ را میخواهد! $[\frac{ab}{4}] = -2$	۹
$\begin{cases} x^2+6x+m=0 \\ -x^2 \pm 2x \mp 3m=0 \end{cases} \Rightarrow 4x+4m=0 \to x=-m$ $\to m^2-6m+m=0 \to m(m-5)=0$ (۰ ✗ ، ۵ ✓) $\to x=-5 \Rightarrow x^2+2x-15=0$, $(x-3)(x+5)$ (۳ ✓ ، ۵- ✓) $x^2+6x+5=0 \to (x+1)(x+5)$ (۱- ✓ ، ۵- ✓) $\Rightarrow 3-(-1) = 4$ (۲)	۱۰

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$$\alpha+\beta=\frac{1}{\alpha\beta} \rightarrow \frac{1-2m}{3}=\frac{3}{2-m} \rightarrow 2m^2-5m-7=0$$

$$(m+1)(2m-7)=0 \rightarrow m=\frac{7}{2} \text{ ی } m=-1$$

$m=-1 \rightarrow 3x^2-3x+3 \rightarrow \Delta<0$ غ‌ق‌ق ✗

$m=\frac{7}{2} \rightarrow 3x^2+9x-\frac{3}{2} \rightarrow \frac{c}{a}<0$ ✓

قطعا ریشه دارد!

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$x^2-x-4=0 \rightarrow S=1 \; , \; P=-4$

$S'=(x_1^3+x_2^3)+(\frac{1}{x_1}+\frac{1}{x_2}) \rightsquigarrow S'=S^3-3SP+\frac{S}{P}=(1)^3-(3)(-2)(1)+\frac{1}{-4}=\frac{121}{4}$

$P'=(x_1x_2)^3+x_2^2+x_1^2+\frac{1}{x_1x_2} \rightarrow P'=P^3+S^2-2P+\frac{1}{P}$

$P'=(-4)^3+1-2(-2)-\frac{1}{4}=-64+9-\frac{1}{4}=\frac{-221}{4}$

$x^2-S'x+P'=x^2-\frac{121}{4}x-\frac{221}{4}$

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α,β ریشه‌های $2ax^2+ax-4$ ، $\alpha+10,\beta+10$ ریشه‌های $2ax^2+ax-4$ هستند!

$S=\underbrace{\alpha+\beta}_{\frac{-a}{2a}}+1=\frac{a}{2} \rightsquigarrow \frac{a}{2}=\frac{1}{2} \rightsquigarrow a=1$

$P=(\alpha+10)(\beta+10)=\alpha\beta+10(\alpha+\beta)+100=\frac{-4}{2a}+\frac{1}{2}(\frac{-a}{2a})+\frac{1}{4}=-\frac{4}{2}-\frac{1}{4}+\frac{1}{4}=\frac{b}{2} \rightarrow b=-4$

$[\frac{ab}{2}]=[\frac{-4}{2}]=[-1,0]=-2$

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$\begin{cases} x^2+ax-2 \rightarrow -\frac{b}{2a}=-\frac{a}{2} \\ -x^2-2x+b \rightarrow -\frac{b}{2a}=\frac{2}{-2} \end{cases} \rightarrow -\frac{a}{2}=-1 \rightarrow a=2 \rightarrow \begin{cases} y=x^2+2x-2 \\ y=1 \end{cases} \rightarrow x^2+2x-3=0$

$\rightarrow x=-3 \rightsquigarrow M(-3,1)$

$\hookrightarrow x=1 \rightsquigarrow N(1,1)$

$-x^2-2x+b \xrightarrow[y=1]{x=-3} -9+6+b=1 \rightarrow b=4 \rightarrow ab=8$