باران عباسی — دهم، فصل ۲

۲۰ آفرین

①

$y=a(x-h)^2+k \to a(x+1)^2+9$

$1=a(2+1)^2+9 \to 1=18a+9 \to -8=16a \to a=-\frac{1}{2}$

$y=-\frac{1}{2}(x+1)^2+9$

$y=-\frac{1}{2}(x^2+1+2x)+9 \to -\frac{1}{2}x^2-x+\frac{17}{2}$ ✓

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②

$2x^2+mx+m+9=0 \qquad \Delta>0$

$\Delta=m^2-4(2)(m+9) \to m^2-8m-48>0 \to (m-12)(m+4)>0$

$m>12$ یا $m<-4$

$\frac{c}{a}=\frac{m+9}{2}>0 \to m+9>0 \to m>-9$

$x_1+x_2 \to \frac{-b}{a}=\frac{-m}{2}>0 \to m<0$

$-9 \quad -4 \quad 0 \quad 12$

$-9<m<-4$ ✓

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③

$3x^2+(2m-1)x+2-m=0 \qquad \frac{c}{a}=\frac{2-m}{3}$

$\frac{-b}{a}=\frac{a}{c} \qquad \frac{-b}{a}=\frac{-2m-1}{3}$

$\to \frac{-2m+1}{3}=\frac{3}{2-m} \to -(2m-1)(2-m)=9$

$-(-2m^2+5m-2)=9 \to 2m^2-5m+2=9 \to 2m^2-5m-7=0$

$\to \sqrt{\Delta}=(-5)^2-4\times2\times(-7)$

$m=\frac{5\pm9}{4} \to m=-1 \;/\; \frac{7}{2}$ ✓

غ ق ق $\Delta<0$

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④

$\frac{-b}{a}=1 \qquad \frac{c}{a}=-4 \qquad (x_1^3+x_2^3)\left(\frac{1}{x_1}+\frac{1}{x_2}\right)$

$(x_1+x_2)^3-3x_1x_2(x_1+x_2) \to 1+12=13$

$\frac{x_2+x_1}{x_1x_2}=\frac{1}{-4}$

$13-\frac{1}{4}=\frac{51}{4}$ جمع

ضرب $\left(x_1^3+\frac{1}{x_2}\right)\left(x_2^3+\frac{1}{x_1}\right) \to x_1^3x_2^3+x_1^2+x_2^2+\frac{1}{x_1x_2} \to -64+9-\frac{1}{4}=\frac{-221}{4}$

$x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2 \to 1+8=9$

$y=x^2-\frac{51}{4}x-\frac{221}{4}$ ✓

②

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⑤

$t=\sqrt[3]{x} \to \left(t^2+\frac{1}{t^2}+1\right)(t^2-1)=2t$

$(t^2-1)t^2+(t^2-1)\frac{1}{t^2}+(t^2-1) \to (t^4-t^2)+\left(1-\frac{1}{t^2}\right)+(t^2-1)$

$t^4-\frac{1}{t^2}=2t \to t^6-2t^3-1=0 \qquad t^3=a$

$a^2-2a-1=0$

$a=\frac{2\pm\sqrt{8}}{2} \to 1\pm\sqrt{2}$

$t^3=1+\sqrt{2} \to t=\sqrt[3]{1+\sqrt{2}}$

$t^3=1-\sqrt{2} \to t=\sqrt[3]{1-\sqrt{2}}$

$x=t^3$

$x=1+\sqrt{2}, \; 1-\sqrt{2}$ ✓

$x_1+x_2=1+\sqrt{2}+1-\sqrt{2}=2$

②

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$\frac{-b}{a} = \frac{a}{3}$ $\frac{c}{a} = \frac{4}{3}$

$x_1 = 3x_2 \longrightarrow x_1 + x_2 = 4x_2 = \frac{a}{3} \longrightarrow x_2 = \frac{a}{12}$ $x_1 = \frac{a}{4}$

$\frac{a}{4} \times \frac{a}{12} = \frac{a^2}{48} \longrightarrow \frac{4}{3} \longrightarrow a^2 = 64 \longrightarrow a = \pm 8$ $8-(-8)=16$

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 $\longrightarrow a > 0$

$x, y < 0$

$ax^2 + (3+2a)x \longrightarrow x(ax+3+2a)$

$y<0$, $x<0 \longrightarrow$ [illegible]

$y>0$ $x<0 \longrightarrow$ [illegible] $\longrightarrow x(ax+2a+3) \geq 0 \longrightarrow x<0$ < $x=0$, $x = \frac{-3-2a}{a}$

$\frac{-3-2a}{a} < 0 \longrightarrow x_2 < 0$

$y>0$ $x_2 < 0 \longrightarrow a>$

$x \longrightarrow -\infty$

$3+2a>0 \longrightarrow a<0$

$a < -\frac{3}{2}$

$-\frac{3}{2} < a < 0$ ①

② $a>0$ ← [illegible]

$① \cap ② = \emptyset$

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$1 = x^2 + ax - 2 \longrightarrow x^2 + ax - 3 = 0$

$1 = x^2 + 2x - b \longrightarrow x^2 + 2x - b + 1 = 0$

$\frac{-a}{2} = \frac{-2}{2} = -1$

$\longrightarrow a = 2$

$1 = x^2 + 2x \longrightarrow (x+3)(x-1) = 0$ < $1 \longrightarrow A(1,1)$, $-3 \longrightarrow B(-3,1)$

$b = 4 \longleftarrow 1 = -9+6+b$

$ab = 8$

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معادله اول $\begin{cases} S = \alpha + 0.5 + \beta + 0.5 = \frac{-1}{2} + 1 \longrightarrow \frac{1}{2} = \frac{a}{2} \longrightarrow a = 1 \\ P = (\alpha + 0.5)(\beta + 0.5) = \frac{-3}{2} \longrightarrow \frac{b}{2} \longrightarrow b = -3 \end{cases}$

معادله دوم $\begin{cases} S = \alpha + \beta = \frac{-1}{2} \\ P = \alpha\beta = -3a \end{cases}$

$\left[\frac{ab}{4}\right] = -1$

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K ← ریشه مشترک

معادله اول $\longrightarrow K^2 + 2K - 3m = 0 \longrightarrow 3m = K^2 + 2K \longrightarrow m = \frac{K^2 + 2K}{3}$

معادله دوم $\longrightarrow K^2 + 6K + m = 0 \longrightarrow m = -(K^2 + 6K)$

$\frac{K^2 + 2K}{3} = -K^2 - 6K$

$\longrightarrow 4K^2 + 20K = 0$

$4K(K+5) = 0$ → $K=0$ غ ق ق (فرض مسئله غیرصفر), $K=-5$

$m = \frac{K^2 + 2K}{3} \longrightarrow \frac{(-5)^2 + 2(-5)}{3} = \frac{25-10}{3} = 5$

$x^2 + 2x - 3m \longrightarrow x^2 - 2x - 15 = 0$

$x_1 = K = -5$ $\frac{-b}{a} = -2$

$x_2 = 3$

$3-(-1) = 4$

$x^2 + 6x + m = x^2 + 6x + 5 \longrightarrow 0 \longrightarrow x_1 = -5$ $\frac{-b}{a} = -6 \longrightarrow x_2 = -1$

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