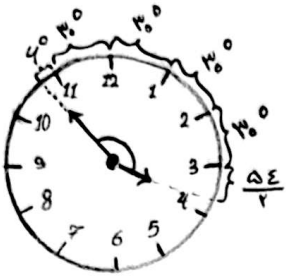
۱) ☆ آذر ۱۹/۲۵

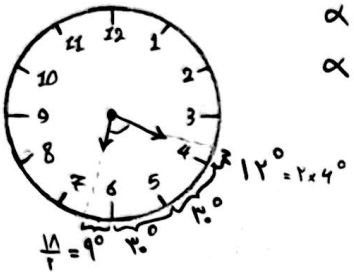


الف) $۳۰+۳۰+۳۰+۳۰+۲۷+۴=\underbrace{۱۵۳^\circ}_{\alpha<۱۸۰^\circ}$ ✓

ب) $\alpha=|۵,۵m-۳۰h|=|(۵,۵\times۵۴)-(۳۰\times۳)|=\underbrace{۲۰۷^\circ}_{\alpha>۱۸۰^\circ}$ ✓

$۳۶۰^\circ-۲۰۷^\circ=\underbrace{۱۵۳^\circ}_{\alpha<۱۸۰^\circ}$ ✓

۲)



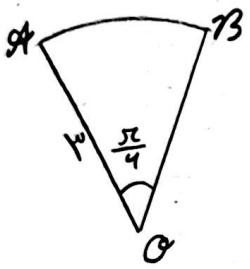
الف) $\alpha<۱۸۰ \rightarrow ۹+۳۰+۳۰+۱۲=\underbrace{۸۱^\circ}_{\alpha<۱۸۰^\circ}$ ✓

$\alpha>۱۸۰ \rightarrow ۳۶۰^\circ-۸۱^\circ=\underbrace{۲۷۹^\circ}_{\alpha>۱۸۰^\circ}$ ✓

ب) $\alpha=|۵,۵m-۳۰h|=|(۵,۵\times۱۸)-(۳\times۹)|=\underbrace{۸۱^\circ}_{\alpha<۱۸۰^\circ}$ ✓

$۳۶۰^\circ-۸۱^\circ=\underbrace{۲۷۹^\circ}_{\alpha>۱۸۰^\circ}$ ✓

۳)

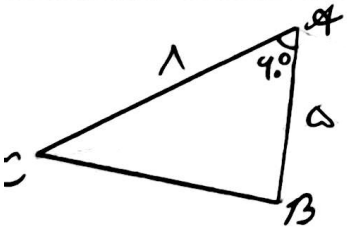


الف) $S_{AOB}=\frac{\alpha}{۲}R^۲=\frac{\frac{\pi}{۴}}{۲}\times۳^۲=\frac{\pi}{۸}\times۹=\frac{۹\pi}{۱۲}=\underbrace{\frac{۳\pi}{۴}}$ ✓

ب) $P_{AOB}=\underbrace{\mathcal{L}}+۲R=\frac{\pi}{۲}+(۲\times۳)=\frac{\pi}{۲}+۶=\underbrace{\frac{۱۲+\pi}{۲}}$ ✓

$\mathcal{L}=R\times\alpha=\frac{۳\pi}{۴}=\frac{\pi}{۲}$

۴)

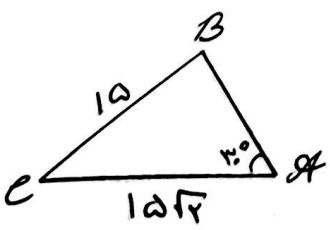


الف) $S_{ABC}=\frac{۱}{۲}\times AC\times AB\times\sin۴۰^\circ=\frac{۱}{۲}\times۸\times۵\times\frac{\sqrt{۳}}{۲}=\underbrace{۱۰\sqrt{۳}}$ ✓

ب) $P_{ABC}=AC+AB+BC=۸+۵+۷=\underbrace{۲۰}$ ✓

$BC=\sqrt{AC^۲+AB^۲-۲AC\cdot AB\cos\underbrace{A}_{۴۰^\circ}}=\sqrt{\underbrace{(۸)^۲}_{۶۴}+\underbrace{(۵)^۲}_{۲۵}-۲\underbrace{(۵\times۸)}_{۴۰}\times\frac{۱}{۲}}=۷$ ✓

۵)



$\hat{B}+\hat{C}=۱۵۰^\circ \rightarrow \hat{A}=۳۰^\circ$

$\frac{\sin۳۰^\circ}{۱۵}=\frac{\sin\hat{B}}{۱۵\sqrt{۲}} \rightarrow \frac{۱}{۲}=\frac{\sin\hat{B}}{۱۵\sqrt{۲}} \rightarrow \frac{۱}{۳۰}=\frac{\sin\hat{B}}{۱۵\sqrt{۲}} \rightarrow \sin\hat{B}=\frac{\sqrt{۲}}{۲} \rightarrow \hat{B}=۴۵^\circ$

$\hat{B}+\hat{C}=۱۵۰^\circ \rightarrow ۴۵+\hat{C}=۱۵۰^\circ \rightarrow \hat{C}=۱۰۵^\circ$ ✓

$\hat{B}=۴۵^\circ=\underbrace{\frac{\pi}{۴}}$ ✓　$\hat{C}=۱۰۵^\circ=\underbrace{\frac{۷\pi}{۱۲}}$ ✓

۶

$$\frac{\tan(\pi-\alpha)+3\tan(\pi+\alpha)}{\tan(2\pi-\alpha)-\tan(2\pi+\alpha)}=\frac{-\tan\alpha+3\tan\alpha}{-\tan\alpha-\tan\alpha}=\frac{2\tan\alpha}{-2\tan\alpha}=-1$$

۷

$\tan\frac{\pi}{12}=a$

$$A=\frac{2\tan75^\circ+\tan10^\circ}{3\tan149^\circ-\tan225^\circ}=\frac{2\overbrace{\tan\left(\frac{\pi}{2}-15^\circ\right)}^{\cot a}+\overbrace{\tan\left(\frac{\pi}{2}+15^\circ\right)}^{-\cot a}}{3\underbrace{\tan(\pi-15^\circ)}_{-\tan a}-\underbrace{\tan\left(\frac{3\pi}{2}-15^\circ\right)}_{+\cot a}}=\frac{2\cot a-\cot a}{-3\tan a-\cot a}$$

۸

$$\frac{\sin x+\cos x}{\sin x-\cos x}+\frac{\sin x-\cos x}{\sin x+\cos x}=3=\frac{(\sin x^2+\cos x^2+2\sin x\cos x)+(\sin x^2+\cos x^2-2\sin x\cos x)}{(\sin x-\cos x)(\sin x+\cos x)}$$

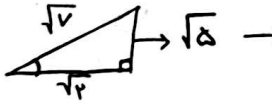
$$\frac{2\sin x^2+2\cos x^2}{\sin x^2-\cos x^2}=\frac{2\overbrace{(\sin x^2+\cos x^2)}^{1}}{\underbrace{\sin x^2-\cos x^2}_{1-\cos x^2}}=\frac{2}{1-2\cos x^2}=3\to 1-2\cos x^2=\frac{3}{2}\to 2\cos x^2=-\frac{1}{2}$$

$$\to\cos x^2=-\frac{1}{5}\quad\to\sin x^2=\frac{4}{5}\quad\to\tan x^2=\frac{\sin x^2}{\cos x^2}=\frac{\frac{4}{5}}{\frac{-1}{5}}=-4$$

۹

$$\frac{\sin^2x-2\cos^2x+1}{\sin^2x+2\cos^2x-1}=5\to\frac{\sin^2x-2+2\sin^2x+1}{-\cos^2x+2\cos^2x}=5\to\frac{3\sin^2x-1}{\cos^2x}=5\to\frac{4-3\cos^2x-1}{\cos^2x}=5$$

$$\to\frac{3-3\cos^2x}{\cos^2x}=5\to\frac{3}{\cos^2x}-3=5\to\frac{3}{\cos^2x}=8\to\cos^2x=\frac{3}{8}\to\cos=\frac{\sqrt{3}}{\sqrt{8}}$$



$$\to\sin x=\frac{\sqrt{5}}{\sqrt{8}}\quad\sin^2x=\frac{5}{8}\quad\Rightarrow\tan^2x=\frac{\sin^2x}{\cos^2x}=\frac{5}{3}$$

۱۰

الف) $\cos(22/5^\circ)\to\cos^2(22/5^\circ)=\frac{1+\cos 2a}{2}=\frac{1+\cos 45^\circ}{2}=\frac{2+\sqrt{2}}{4}=\frac{2+\sqrt{2}}{4}$

$$\to\cos(22/5^\circ)=\frac{\sqrt{2+\sqrt{2}}}{2}$$

ب) $\sin(67/5^\circ)\to\sin^2(67/5^\circ)=\frac{1-\cos^2a}{2}=\frac{1-\cos 135^\circ}{2}=\frac{2+\sqrt{2}}{4}=\frac{2+\sqrt{2}}{4}$

$$\to\sin(67/5^\circ)=\frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\frac{\pi}{12} = \left(\frac{\pi}{4}\right) \times \frac{1}{3} = \frac{30^\circ}{3} = 15^\circ$$

$$A = \frac{2\tan\left(\frac{\pi}{2} - 15^\circ\right) + \tan\left(\frac{\pi}{2} + 15^\circ\right)}{3\tan(\pi - 15^\circ) - \tan\left(\frac{3\pi}{2} - 15^\circ\right)} = \frac{2\cot 15^\circ - \cot 15^\circ}{-3\tan 15^\circ - \cot 15^\circ}$$

$$A = \frac{\cot 15^\circ}{-3\tan 15^\circ - \cot 15^\circ} \xrightarrow{\tan 15^\circ = a} \frac{\frac{1}{a}}{-3a - \frac{1}{a}} = \frac{-1}{3a^2 + 1}$$

$$\frac{(\sin\alpha + \cos\alpha)^2 + (\sin\alpha - \cos\alpha)^2}{\sin^2\alpha - \cos^2\alpha} = \frac{2\overbrace{(\sin^2\alpha + \cos^2\alpha)}^{1}}{\sin^2\alpha - \cos^2\alpha} = 2$$

$$2 = 3\sin^2\alpha - 3\cos^2\alpha \rightarrow 2 = 3(1 - \cos^2\alpha) - 3\cos^2\alpha$$

$$4\cos^2\alpha = 1 \rightarrow \cos^2\alpha = \frac{1}{4} \rightarrow 1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \rightarrow \boxed{\tan^2\alpha = 3}$$