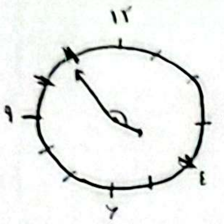
۱-

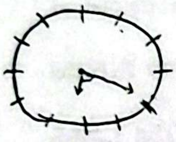
الف)



$4(30°) + 6° + 27° = 126 + 27 = 153°$ ($54 \div 2$)

ب) $\alpha = |5.5M - 30H| = |\frac{11}{2}\times 54 - 30\times 3| = |297 - 90| = 207$ $\quad 360 - 207 = 153$

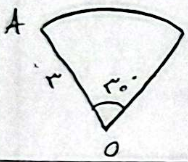
۲- الف)



$2(30°) + 2(6°) + 9 = 81°$ ($\frac{18}{2}$)

ب) $\alpha = |5.5M - 30H| = |\frac{11}{2}\times 18 - 30\times 6| = |99 - 180| = 81°$

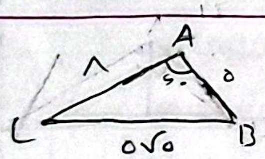
۳- الف)



فرمول: $S = \frac{\alpha}{2}R^2 = 15\times 9 = 135$

ب) فرمول: $P = \alpha R + 2R = 30\times 3 + 6 = 90 + 6 = 96$

۴- الف)



$S = \frac{1}{2}\times 5\times 8\times \frac{\sqrt{3}}{2} = 10\sqrt{3}$

ب) $BC = \sqrt{c^2 + b^2 + 2bc\cos 60°} = \sqrt{25 + 64 + 2\times 40\times \frac{1}{2}} = \sqrt{85 + 40} = \sqrt{125} = 5\sqrt{5}$

$P = 13 + 5\sqrt{5}$

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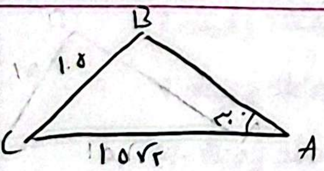

$\hat{B} + \hat{C} = 150$

$\frac{15}{\frac{1}{2}} = \frac{15\sqrt{2}}{\sin B} = \frac{AB}{\sin C}$

$\sin\hat{B} = \frac{\sqrt{2}}{2} = 45°$

$B = 45 = \frac{\pi}{4}$

$C = 150 - 45 = 105 \to 105\times\frac{\pi}{180} = \frac{7\pi}{12}$

۶-

$$\frac{\tan(\pi-\alpha) + 3\tan(\pi+\alpha)}{\tan(2\pi-\alpha) - \tan(2\pi+\alpha)} = \frac{-\tan(\alpha) + 3\tan(\alpha)}{-\tan(\alpha) - \tan(\alpha)} = +2\tan(\alpha)$$

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$\tan\frac{\pi}{12} = \tan 15° = a$

$$A = \frac{2\tan 75° + \tan 105°}{3\tan 165° - \tan 255°} = \frac{2\tan(\frac{\pi}{2} - 15°) + \tan(\frac{\pi}{2} + 15°)}{3\tan(\pi - 15°) - \tan(\frac{3\pi}{2} - 15°)}$$

$$= \frac{2\cot(15°) - \cot(15°)}{-3\tan(15°) - \cot(15°)} = \frac{\cot 15°}{-3\tan 15° - \cot 15°} = \frac{\frac{1}{a}}{-3a - \frac{1}{a}} = \frac{\frac{1}{a}}{-\frac{3a^2+1}{a}} = -\frac{1}{3a^2+1}$$

۸-

$$\frac{(\sin x+\cos x)^2+(\sin x-\cos x)^2}{\sin^2 x-\cos^2 x}=2$$

$$\frac{\sin^2 x+\cos^2 x+2\sin x\cos x+\sin^2 x+\cos^2 x-2\sin x\cos x}{\sin^2 x-\cos^2 x}=2$$

$$\frac{2(\sin^2 x+\cos^2 x)}{\sin^2 x-\cos^2 x}=\frac{2}{\sin^2 x-\cos^2 x}=2\rightarrow\frac{2}{1-2\cos^2 x}=2\rightarrow 2-4\cos^2 x=2$$

$$1=4\cos^2 x$$

$$\boxed{\frac{1}{4}=\cos^2 x}$$

$$\tan^2 x+1=\frac{1}{\cos^2 x}\Rightarrow \tan^2 x=4-1$$

$$\boxed{\tan^2 x=3}$$

۹-

$$\frac{\sin^2 x-2\cos^2 x+1}{\sin^2 x+2\cos^2 x-1}=2\rightarrow\frac{1-\cos^2 x-2\cos^2 x+1}{1+\cos^2 x-1}=\frac{2-3\cos^2 x}{\cos^2 x}=4$$

$$\Rightarrow 4\cos^2 x=2-3\cos^2 x$$

$$7\cos^2 x=2\rightarrow\boxed{\cos^2 x=\frac{2}{7}}$$

$$\tan^2 x+1=\frac{1}{\cos^2 x}\rightarrow\tan^2 x=\frac{7}{2}-1=\frac{5}{2}$$

۱۰-

فرمول طلایی:

$$\cos^2\alpha=\frac{1+\cos 2\alpha}{2}\rightarrow\cos 22{,}5^2=\frac{1+\cos 45^\circ}{2}$$

$$\cos 22{,}5^2=\frac{1+\frac{\sqrt{2}}{2}}{2}=\frac{2+\sqrt{2}}{4}\rightarrow\boxed{\cos 22{,}5^\circ=\frac{\sqrt{2+\sqrt{2}}}{2}=\sin 67{,}5^\circ}$$