پاسخ تشریحی امتحان هفته B

(۱ الف) $\tan(۳\pi - \frac{\pi}{۴}) + \sin(۴\pi - \frac{\pi}{۴}) \times \cos(۳\pi + \frac{\pi}{۴})$

$= -\tan(\frac{\pi}{۴}) + (-\sin(\frac{\pi}{۴}))(-\cos(\frac{\pi}{۴})) = -۱ + \frac{۱}{۲} = -\frac{۱}{۲}$

ب) $\tan(۳\pi - \frac{\pi}{۶}) \times \sin(۴\pi - \frac{\pi}{۳}) + \cos(۳\pi + \frac{\pi}{۳}) \rightarrow (-\tan(\frac{\pi}{۶}) \times -\sin(\frac{\pi}{۳}))$

$- \cos(\frac{\pi}{۳}) = \frac{۱}{۲} - \frac{۱}{۲} = ۰$

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(۲) عبارت بر $\sin x$ تقسیم $\Rightarrow \frac{۳\cot x - ۱}{\cot x + ۱} = \frac{۳(۴) - ۱}{۴ + ۱} = \frac{۱۱}{۵} = ۲{,}۲$

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(۳) $\sin^۲\alpha + \cos^۲\alpha = ۱ \rightarrow (۲\cos\alpha)^۲ + \cos^۲\alpha = ۱ \rightarrow ۴\cos^۲\alpha + \cos^۲\alpha = ۱$

$۵\cos^۲\alpha = ۱ \rightarrow \cos^۲\alpha = \frac{۱}{۵} \rightarrow \cos\alpha$ ↗ $+\frac{۱}{\sqrt{۵}}$ ↘ $-\frac{۱}{\sqrt{۵}}$ قابل قبول

(۴ الف) $\cos(۲\pi + \frac{۳\pi}{۴} + \alpha) \Rightarrow \cos(\frac{۳\pi}{۴} + \alpha) \rightarrow \cos\frac{۳\pi}{۴} \times \cos\alpha - \sin\frac{۳\pi}{۴} \times \sin\alpha$

$\cos\alpha = -\sqrt{۱ - \sin^۲\alpha} \rightarrow \cos\alpha = -\sqrt{۱ - \frac{۱}{۵۰}} = \frac{-۷}{\sqrt{۵۰}} = \frac{-۷}{۵\sqrt{۲}} \rightarrow (-\frac{\sqrt{۲}}{۲} \times \frac{-۷}{۵\sqrt{۲}}) - \frac{\sqrt{۲}}{۲} \times \frac{\sqrt{۲}}{۱۰} = \frac{۷ - ۱}{۱۰} = \frac{۶}{۱۰} = \frac{۳}{۵}$

ب) $\sin(۲\pi + \frac{۵\pi}{۴} + \alpha) = \sin(\frac{۵\pi}{۴} + \alpha) \rightarrow \sin\frac{۵\pi}{۴} \times \cos\alpha + \sin\alpha \times \cos\frac{۵\pi}{۴}$

$-\frac{\sqrt{۲}}{۲} \times \frac{۷}{۵\sqrt{۲}} + \frac{۱}{۵\sqrt{۲}} \times -\frac{\sqrt{۲}}{۲} = -\frac{۸}{۱۰} = -\frac{۴}{۵}$

$-\frac{۷}{۱۰}$ ، $-\frac{۱}{۱۰}$

$\sin\alpha = \frac{۱}{۵\sqrt{۲}}$

$\tan^۲ + ۱ = \frac{۱}{\cos^۲} \Rightarrow \cos = \frac{۷}{۵\sqrt{۲}}$

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(۵) $\sin^۲ x = ۱ - \cos^۲ x \longrightarrow ۲(۱ - \cos^۲ x) + \cos^۲ x = \frac{۴}{۳}$

$\cos^۲ x = \frac{۲}{۳}$ $\quad ۱ + \tan^۲ x = \frac{۱}{\cos^۲ x} \Rightarrow ۱ + \tan^۲ x = \frac{۳}{۲} \rightarrow \tan^۲ x = \frac{۱}{۲}$

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(۶) $S_{مثلث} = \frac{۱}{۲} \times ۶ \times \sqrt{۴} \times \sin\alpha = ۴{,}۵ \rightarrow ۳\sqrt{۳}\sin\alpha = ۴{,}۵ \quad \frac{۹}{۲}\sin\alpha \quad \sin\alpha = \frac{۴{,}۵}{۳\sqrt{۳}} = \frac{\sqrt{۳}}{۲}$

$\longrightarrow \alpha$ ↗ ۶۰° ↘ ۱۲۰° $\rightarrow \frac{۱۲۰^\circ}{۶۰^\circ} = ۲$

(۷)

$\left(\frac{1}{2} \times 3m \times 2a \times \sin 150^\circ\right) \times 2 = 5\Sigma$ ($\sin 150^\circ = \frac{1}{2}$)

$3m \times 2 = 5\Sigma \longrightarrow m = \pm\sqrt{15}$

$b_{\Theta} = (3\sqrt{15} + 2\sqrt{15}) \times 2 = 10\sqrt{15} = 20\sqrt{3}$

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(۸)

$S_{ABC} = \frac{1}{2} \times a \times V \times \sin\hat{A} = \frac{3a}{2} \sin\hat{A} = 15a \sin\hat{A}$

$S_{ADE} = \frac{1}{2} \times \Sigma \times V \times \sin\hat{A} = 12 \sin\hat{A}$

$3a \sin\hat{A} = 15V$

$\Longrightarrow \sin\hat{A} = \frac{1}{2} \longrightarrow \tan 30^\circ = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$\hat{A} = 30^\circ$ (رد 150°)

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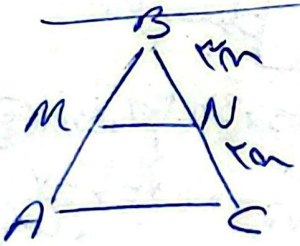
(۹)



$AB \times 2m \times \frac{1}{2} \times \sin\hat{B} = (\frac{1}{2} \times \sin\hat{B} \times 3m \times BM) \times 2$

$AB = BM + AM$

$2BM + 2AM = 9BM$

$2AM = 7BM \quad \frac{BM}{AM} = \frac{2}{7}$

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(۹)

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} \longrightarrow \frac{|\sin\alpha|}{\cos\alpha} = \frac{-\sin\alpha}{\cos\alpha} \longrightarrow \sin\alpha < 0$

$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1 + \sin\alpha}{|\cos\alpha|} \longrightarrow \tan\alpha = \frac{-\sin\alpha}{|\cos\alpha|}$

$|\cos\alpha|$

$\checkmark \tan\alpha = \tan\alpha \leftarrow \cos\alpha < 0$

$\times \tan\alpha = -\tan\alpha \leftarrow \cos\alpha > 0$

$\Longrightarrow \cos\alpha < 0, \sin\alpha < 0$ ربع سوم