نام و نام خانوادگی: تانیا ترک ترابی سری: ۱۸۰ گروه: دهم یکشنبه

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الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4}\cos\frac{13\pi}{4} = \tan(2\pi+\frac{3}{4}\pi) + \sin(4\pi-\frac{\pi}{4}) \times \cos(3\pi+\frac{\pi}{4})$

$= -\tan\frac{3\pi}{4} + (\sin\frac{3\pi}{4} \times \cos\frac{\pi}{4}) = -1 + \underbrace{(-\frac{\sqrt{2}}{2} \times -\frac{\sqrt{2}}{2})}_{\frac{2}{4}} = -\frac{1}{2}$

ب) $\tan\frac{17\pi}{4}\sin\frac{11\pi}{3} + \cos\frac{10\pi}{4} = \tan(2\pi+\frac{5\pi}{4}) \times \sin(4\pi-\frac{\pi}{3}) + \cos(2\pi+\frac{4\pi}{4}) =$

$-\tan\frac{5\pi}{4} \times -\sin\frac{\pi}{3} + \frac{4\pi}{3} = (-\frac{\sqrt{3}}{3} \times -\frac{\sqrt{3}}{2}) - \frac{1}{2} = \frac{1}{2} - \frac{1}{2} = 0$

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$$\frac{3\cos x - \sin x}{\cos x + \sin x} \xrightarrow{\times \sin x} \frac{\frac{3\cos x}{\sin x} - \frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}} = \frac{3\cot x - 1}{\cot x + 1} = \frac{12-1}{4+1} = \frac{11}{5}$$

$\cot x = 4$

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$\sin a = 2\cos a$

$\cos^2 a = 1 - \sin^2 a \Rightarrow \cos^2 a = 1 - 4\cos^2 a \Rightarrow 5\cos^2 a = 1 \Rightarrow \cos^2 a = \frac{1}{5}$

$\Rightarrow \cos a = \begin{cases} -\frac{1}{\sqrt{5}} \checkmark = -\frac{\sqrt{5}}{5} \\ +\frac{1}{\sqrt{5}} \times \end{cases}$

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الف)

$\sin a = \frac{\sqrt{2}}{10}$ $\cos(\frac{11\pi}{4}+a) = \cos(2\pi+\frac{3}{4}\pi+a) = -\cos\frac{3\pi}{4}+a$

$\cos^2 a = 1 - \sin^2 a \Rightarrow \cos^2 a = 1 - \frac{2}{100} \Rightarrow \cos^2 a = \frac{49}{50} \Rightarrow \cos a = \begin{cases} \frac{7}{\sqrt{50}} = \frac{7\sqrt{2}}{10} \times \\ -\frac{7}{\sqrt{50}} = \frac{7\sqrt{2}}{10} \checkmark \end{cases}$

$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ $\cos(\frac{11\pi}{4}+a) = \cos\frac{11\pi}{4}\cos a - \sin\frac{11\pi}{4}\sin a \Rightarrow$

$(-\frac{\sqrt{2}}{2})(-\frac{7\sqrt{2}}{10}) - (\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{10}) = \frac{12}{20} = \frac{3}{5}$

$\sin^2 a = 1 - \frac{49}{50} = \frac{1}{50} = \frac{\sqrt{2}}{10}$

ب)

$\sin(\frac{13\pi}{4}+a) = \sin(3\pi+\frac{\pi}{4}+a) = -\sin\frac{\pi}{4}+a = -\frac{\sqrt{2}}{2}+a$

$\tan a = \frac{1}{7} \Rightarrow \frac{\sin a}{\cos a} = \frac{1}{7}$ $\sin(\frac{13\pi}{4}+a) = \sin(\frac{13\pi}{4})\cos a + \cos(\frac{13\pi}{4})\sin a =$

$1+\tan^2 a = \frac{1}{\cos^2 a} \Rightarrow 1 + \frac{1}{49} = \frac{1}{\cos^2 a} \Rightarrow 49 = 50\cos^2 a \Rightarrow \cos^2 a = \frac{49}{50} \Rightarrow \cos a = \frac{7\sqrt{2}}{10} \Rightarrow \sin a = \frac{\sqrt{2}}{10}$

$-\frac{\sqrt{2}}{2}(-\frac{7\sqrt{2}}{10} - \frac{\sqrt{2}}{10}) = \frac{50}{49} \rightarrow \frac{4}{5}$

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$2\sin^2 x + \cos^2 x = \frac{4}{3} \xrightarrow{\div \cos^2 x} \frac{2\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{4}{3} \Rightarrow 2\tan^2 x + 1 = \frac{4}{3} \Rightarrow$

$2\tan^2 x = \frac{1}{3} \Rightarrow \tan^2 x = \frac{1}{6}$

$۴\sqrt{۵} = \frac{۱}{۲} \times ۹ \times \sqrt{۲} \times \sin\alpha \Rightarrow ۴\sqrt{۵} = ۴\sqrt{۲} \times \sin\alpha \Rightarrow \frac{۴\sqrt{۵}}{۴\sqrt{۲}} \times \frac{۱}{۲\sqrt{۲}} = \sin\alpha \Rightarrow \frac{\sqrt{۲}}{۲} = \sin A$

$a^۲ = b^۲ + c^۲ - ۲bc\cos A \Rightarrow a^۲ = ۴۹ - ۱۲\sqrt{۲}\cos A$

① $A_۱ = ۹۰^\circ$ $\cos ۹۰^\circ = \frac{۱}{۲}$ $a_۱^۲ = ۴۹ - ۱۲\sqrt{۲} \times \frac{۱}{۲} = ۴۹ - ۶\sqrt{۲}$

② $A_۲ = ۱۲۰^\circ$ $\cos ۱۲۰^\circ = -\frac{۱}{۲}$ $a_۲^۲ = ۴۹ - ۱۲\sqrt{۲}\left(-\frac{۱}{۲}\right) = ۴۹ + ۶\sqrt{۲}$

$\frac{a_{max}}{a_{min}} = \frac{\sqrt{۴۹ + ۶\sqrt{۲}}}{\sqrt{۴۹ - ۶\sqrt{۲}}} \Rightarrow \frac{a_{max}^۲}{a_{min}^۲} = \frac{(۴۹+۶\sqrt{۲})(۴۹+۶\sqrt{۲})}{(۴۹-۶\sqrt{۲})(۴۹+۶\sqrt{۲})} = \frac{(۴۹+۶\sqrt{۲})^۲}{۴۹^۲ - (۶\sqrt{۲})^۲} \Rightarrow \frac{a_{max}}{a_{min}} = \sqrt{\frac{۴۹+۶\sqrt{۲}}{۴۹-۶\sqrt{۲}}}$

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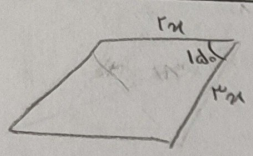

$\frac{۱}{۲}ab = ۵۴ \Rightarrow ab = ۱۰۸$ $\frac{a}{b} = \frac{۲}{۳} \Rightarrow a = \frac{۲}{۳}b$

$\left(\frac{۲}{۳}b\right)b = ۱۰۸ \Rightarrow \frac{۲}{۳}b^۲ = ۱۰۸ \Rightarrow b^۲ = ۱۶۲ \Rightarrow b = ۹\sqrt{۲}$

$a = \frac{۲}{۳}(۹\sqrt{۲}) = ۶\sqrt{۲}$

$P = ۲(a+b) = ۲(۶\sqrt{۲} + ۹\sqrt{۲}) = ۳۰\sqrt{۲}$

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$S(ADE) = \frac{۱}{۲} \times AE \times AD \times \sin A = \frac{۱}{۲} \times ۴ \times ۷ \times \sin\alpha \Rightarrow ۱۴\sin\alpha$

$S(ABC) = \frac{۱}{۲} \times AB \times AC \times \sin A = \frac{۱}{۲} \times ۵ \times ۷ \times \sin\alpha = \frac{۳۵}{۲}\sin\alpha = ۱۷/۵\sin\alpha$

$۱۷/۵\sin\alpha - ۱۴\sin\alpha = ۱۱/۷۵ \Rightarrow ۳/۵\sin\alpha = ۱۱/۷۵ \Rightarrow \sin\alpha = \frac{۱۱/۷۵}{۳/۵} \Rightarrow \sin\alpha = \frac{۱}{۲}$

$\cos\alpha = \frac{\sqrt{۳}}{۲}$

$\tan\alpha = \frac{\frac{۱}{۲}}{\frac{\sqrt{۳}}{۲}} = \frac{۱}{\sqrt{۳}} = \frac{\sqrt{۳}}{۳}$

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$\frac{\frac{۱}{۲}BM \times BN \times \sin B}{\frac{۱}{۲}AB \times BC \times \sin B} = \frac{۱}{۳}$ $\frac{BN \times BM}{AB \times BC} = \frac{۱}{۳}$ $\frac{BN}{BC} = \frac{۳}{۵}$

$\frac{BM}{AB} \times \frac{BN}{BC} = \frac{۱}{۳}$ $\frac{BM}{AB} \times \frac{۳}{۵} = \frac{۱}{۳} \Rightarrow \frac{BM}{AB} = \frac{۵}{۹}$ $\frac{AM}{AB} = ۱ - \frac{BM}{AB} = ۱ - \frac{۵}{۹} = \frac{۴}{۹}$

$\frac{BN}{AM} = \frac{BN}{BC} \times \frac{BC}{AB} \times \frac{AB}{AM} \Rightarrow \frac{BN}{AM} = \frac{۳}{۵} \times \frac{BC}{AB} \times \frac{۱}{\frac{۴}{۹}} \Rightarrow \frac{BN}{AM} = \frac{۲۷}{۲۰} \times \frac{BC}{AB} = ۱/۳۵$

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ر.ت.ص ۳

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{۱}{\cot\alpha} \Rightarrow \frac{|\sin\alpha|}{\cos\alpha} = -\frac{\sin\alpha}{\cos\alpha} \rightarrow$ ناحیه ۲ یا ۴

$\frac{۱}{\sqrt{\cos^۲\alpha}} - \tan\alpha = \frac{۱+\sin\alpha}{|\cos\alpha|} \Rightarrow -\tan\alpha = \frac{۱+\sin\alpha}{|\cos\alpha|} - \frac{۱}{|\cos\alpha|} \Rightarrow$

$-\tan\alpha = \frac{۱+\sin\alpha - ۱}{|\cos\alpha|} \Rightarrow -\tan\alpha = \frac{\sin\alpha}{|\cos\alpha|} \rightarrow$ ناحیه ۳

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