نام و نام خانوادگی: تانیا ترک ترابی — سری: ۱۸ — ۱۸,۵ — گروه: دهم یکشنبه

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الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4}\cos\frac{13\pi}{4} = \tan(2\pi + \frac{3}{4}\pi) + \sin(4\pi - \frac{\pi}{4}) \times \cos(3\pi + \frac{\pi}{4})$
$= -\tan\frac{3\pi}{4} + (\sin\frac{3\pi}{4} \times \cos\frac{\pi}{4}) = -1 + \underbrace{(-\frac{\sqrt{2}}{2} \times -\frac{\sqrt{2}}{2})}_{\frac{2}{4}} = -\frac{1}{2}$ ✓

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ب) $\tan\frac{17\pi}{4}\sin\frac{11\pi}{3} + \cos\frac{10\pi}{3} = \tan(2\pi + \frac{5\pi}{4}) \times \sin(4\pi - \frac{\pi}{3}) + \cos(2\pi + \frac{4\pi}{3}) =$
$-\tan\frac{5\pi}{4} \times -\sin\frac{\pi}{3} + \frac{4\pi}{3} = (-\frac{\sqrt{3}}{3} \times -\frac{\sqrt{3}}{2}) - \frac{1}{2} = \frac{1}{2} - \frac{1}{2} = 0$ ✓

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$\frac{3\cos x - \sin x}{\cos x + \sin x} \xrightarrow{\times \sin x} \frac{\frac{3\cos x}{\sin x} - \frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}} = \frac{3\cot x - 1}{\cot x + 1} = \frac{12-1}{4+1} = \frac{11}{5}$ ✓

$\cot x = 4$

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$\sin a = 2\cos a$

$\cos^2 a = 1 - \sin^2 a \Rightarrow \cos^2 a = 1 - 4\cos^2 a \Rightarrow 5\cos^2 a = 1 \Rightarrow \cos^2 a = \frac{1}{5}$

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$\Rightarrow \cos a = -\frac{1}{\sqrt{5}}$ ✓ $= -\frac{\sqrt{5}}{5}$ ✓ ; $+\frac{1}{\sqrt{5}}$ ✗

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الف) $\sin a = \frac{\sqrt{2}}{10}$ $\qquad \cos(\frac{11\pi}{4} + a) = \cos(2\pi + \frac{3}{4}\pi + a) = -\cos\frac{3\pi}{4} + a$

$\cos^2 a = 1 - \sin^2 a \Rightarrow \cos^2 a = 1 - \frac{2}{100} \Rightarrow \cos^2 a = \frac{49}{50} \Rightarrow \cos a = \frac{7}{\sqrt{50}} = \frac{7\sqrt{2}}{10}$ ✗ ; $-\frac{7}{\sqrt{50}} = -\frac{7\sqrt{2}}{10}$ ✓

$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ $\qquad \cos(\frac{11\pi}{4} + a) = \cos\frac{11\pi}{4}\cos a - \sin\frac{11\pi}{4}\sin a \Rightarrow$

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$(-\frac{\sqrt{2}}{2})(-\frac{7\sqrt{2}}{10}) - (\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{10}) = \frac{12}{20} = \frac{3}{5}$ ✓ $\qquad \boxed{\sin^2 a = 1 - \frac{49}{50} = \frac{1}{50} = \frac{\sqrt{2}}{10}}$ ✓

ب) $\sin(\frac{13\pi}{4} + a) = \sin(3\pi + \frac{\pi}{4} + a) = -\sin\frac{\pi}{4} + a = -\frac{\sqrt{2}}{2} + a$

$\tan a = \frac{1}{7} \Rightarrow \frac{\sin a}{\cos a} = \frac{1}{7}$ $\qquad \sin(\frac{13\pi}{4} + a) = \sin(\frac{13\pi}{4})\cos a + \cos(\frac{13\pi}{4})\sin a =$

$1 + \tan^2 a = \frac{1}{\cos^2 a} \Rightarrow 1 + \underbrace{\frac{1}{49}}_{\frac{50}{49}} = \frac{1}{\cos^2 a} \Rightarrow 49 = 50\cos^2 a \Rightarrow \cos^2 a = \frac{49}{50} \Rightarrow \cos a = \frac{7\sqrt{2}}{10} \Rightarrow \sin a = \frac{\sqrt{2}}{10}$

$-\frac{\sqrt{2}}{2}(-\frac{7\sqrt{2}}{10} - \frac{\sqrt{2}}{10}) = \frac{50}{49} \rightarrow \cancel{\frac{4}{5}}$

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$2\sin^2 x + \cos^2 x = \frac{4}{3} \xrightarrow{\div \cos^2 x} \frac{2\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{4}{3} \Rightarrow 2\tan^2 x + 1 = \frac{4}{3} \Rightarrow$

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$2\tan^2 x = \frac{1}{3} \Rightarrow \tan^2 x = \frac{1}{6}$

$۴/۵ = \frac{۱}{۲} \times ۶ \times \sqrt{۳} \times \sin x \Rightarrow ۴/۵ = ۳\sqrt{۳} \times \sin x \Rightarrow \frac{\cancel{۴۵}^{۳}}{\cancel{۱۰}_{۲}} \times \frac{۱}{\cancel{۳}_{۱}\sqrt{۳}} = \sin x \Rightarrow \frac{\sqrt{۳}}{۲} = \sin A$

$a^۲ = b^۲ + c^۲ - ۲bc\cos A \Rightarrow a^۲ = ۳۹ - ۱۲\sqrt{۳}\cos A$

$a_۲^۲ = ۳۹ - ۱۲\sqrt{۳}(-\frac{۱}{۲}) = ۳۹ + ۶\sqrt{۳}$

① $A_۱ = ۶۰^\circ \quad \cos ۶۰^\circ = \frac{۱}{۲} \quad a_۱^۲ = ۳۹ - ۱۲\sqrt{۳} \times \frac{۱}{۲} = ۳۹ - ۶\sqrt{۳}$ ② $A_۲ = ۱۲۰^\circ \quad \cos ۱۲۰^\circ = -\frac{۱}{۲}$

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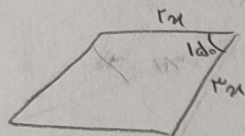
$\frac{a_{max}}{a_{min}} = \frac{\sqrt{۳۹ + ۶\sqrt{۳}}}{\sqrt{۳۹ - ۶\sqrt{۳}}} \Rightarrow \frac{a_{max}^۲}{a_{min}^۲} = \frac{(۳۹ + ۶\sqrt{۳})(۳۹ + ۶\sqrt{۳})}{(۳۹ - ۶\sqrt{۳})(۳۹ + ۶\sqrt{۳})} = \frac{(۳۹ + ۶\sqrt{۳})^۲}{۳۹^۲ - (۶\sqrt{۳})^۲} \Rightarrow \frac{a_{max}}{a_{min}} = \sqrt{\frac{۳۹ + ۶\sqrt{۳}}{۳۹ - ۶\sqrt{۳}}}$ ۶



$\frac{۱}{۲}ab = ۵۴ \Rightarrow ab = ۱۰۸ \quad \frac{a}{b} = \frac{۲}{۳} \Rightarrow a = \frac{۲}{۳}b$

$(\frac{۲}{۳}b)b = ۱۰۸ \Rightarrow \frac{۲}{۳}b^۲ = ۱۰۸ \Rightarrow b^۲ = ۱۶۲ \Rightarrow b = ۹\sqrt{۲}$

$a = \frac{۲}{۳}(۹\sqrt{۲}) = ۶\sqrt{۲}$

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$P = ۲(a+b) = ۲(۶\sqrt{۲} + ۹\sqrt{۲}) = ۳۰\sqrt{۲}$ ✓ ۷

$S(ADE) = \frac{۱}{۲} \times AE \times AD \times \sin A = \frac{۱}{۲} \times ۴ \times ۷ \times \sin a \Rightarrow ۱۴\sin a$

$S(ABC) = \frac{۱}{۲} \times AB \times AC \times \sin A = \frac{۱}{۲} \times ۵ \times ۷ \times \sin a = \frac{۳۵}{۲}\sin a = ۱۷/۵ \sin a$

$۱۷/۵ \sin a - ۱۴\sin a = ۱/۷۵ \Rightarrow ۳/۵ \sin a = ۱/۷۵ \Rightarrow \sin a = \frac{۱/۷۵}{۳/۵} \Rightarrow \sin a = \frac{۱}{۲}$ ۸

$\cos a = \frac{\sqrt{۳}}{۲}$

$\tan a = \frac{\frac{۱}{۲}}{\frac{\sqrt{۳}}{۲}} = \frac{۱}{\sqrt{۳}} = \frac{\sqrt{۳}}{۳}$ ✓

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$\frac{\cancel{\frac{۱}{۲}} BM \times BN \times \cancel{\sin B}}{\cancel{\frac{۱}{۲}} AB \times BC \times \cancel{\sin B}} = \frac{۱}{۳} \quad \frac{BN \times BM}{AB \times BC} = \frac{۱}{۳} \quad \frac{BN}{BC} = \frac{۳}{۵}$

۱/۷۵

$\frac{BM}{AB} \times \frac{BN}{BC} = \frac{۱}{۳} \quad \frac{BM}{AB} \times \frac{۳}{۵} = \frac{۱}{۳} \Rightarrow \frac{BM}{AB} = \frac{۵}{۹} \quad \frac{AM}{AB} = ۱ - \frac{BM}{AB} = ۱ - \frac{۵}{۹} = \frac{۴}{۹}$ ۹

$\frac{BN}{AM} = \frac{BN}{BC} \times \frac{BC}{AB} \times \frac{AB}{AM} \Rightarrow \frac{BN}{AM} = \frac{۳}{۵} \times \frac{BC}{AB} \times \frac{۱}{\frac{۴}{۹}} \Rightarrow \frac{BN}{AM} = \frac{۲۷}{۲۰} \times \frac{\cancel{BC}^{\frac{۳}{۵}}}{\cancel{AB}} = ۰/۸۱$

در ناحیه ۳

$\frac{|\sin a|}{\cos a} = -\frac{۱}{\cot a} \Rightarrow \frac{|\sin a|}{\cos a} = -\frac{\sin a}{\cos a} \xrightarrow{?}$ ناحیه ۳ یا ۴

$\frac{۱}{\sqrt{\cos^۲ a}} - \tan a = \frac{۱ + \sin a}{|\cos a|} \Rightarrow -\tan a = \frac{۱ + \sin a}{|\cos a|} - \frac{۱}{|\cos a|} \Rightarrow$ ۱۰

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$-\tan a = \frac{\cancel{۱+}\sin a \cancel{-۱}}{|\cos a|} \Rightarrow -\tan a = \frac{\sin a}{|\cos a|} \rightarrow$ ✓ ناحیه ۳

۴- ب

$\tan\alpha = \frac{1}{7} \rightarrow 1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \rightarrow 1+\frac{1}{49} = \frac{1}{\cos^2\alpha}$

$\cos^2\alpha = \frac{49}{50} \xrightarrow[\cos\alpha>0]{\text{ناحیه اول}} \cos\alpha = \frac{7}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{10}$

$\sin^2\alpha = 1-\frac{49}{50} = \frac{1}{50} \xrightarrow[\sin\alpha>0]{\text{ناحیه اول}} \sin\alpha = \frac{1}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{10}$

$\sin(3\pi + \frac{\pi}{4} + \alpha) = -\sin(\frac{\pi}{4}+\alpha) = -(\frac{\sqrt{2}}{2}\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha)$

$-\frac{\sqrt{2}}{2}(\frac{7\sqrt{2}}{10} + \frac{\sqrt{2}}{10}) = \frac{-1}{\cancel{\sqrt{2}}} \times \frac{8\cancel{\sqrt{2}}}{10} = \boxed{-\frac{4}{5}}$

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$6 \times \sqrt{3} \times \sin\alpha \times \frac{1}{2} = \frac{9}{2} \rightarrow \sin\alpha = \frac{9}{6\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{18} = \frac{\sqrt{3}}{2}$

مثلث $\rightarrow 0<\alpha<\pi$ $\rightarrow \alpha = \frac{\pi}{3}$ یا 60°

$\rightarrow \alpha = \frac{2\pi}{3}$ یا 120° $\rightarrow \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = \boxed{2}$

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$\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3} \rightarrow \sin^2 x = \frac{1}{3}$

$\cos^2 x = 1 - \frac{1}{3} = \frac{2}{3}$ ، $1+\tan^2 x = \frac{1}{\frac{2}{3}} \rightarrow \tan^2 x = \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$

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$\frac{S_{ABC}}{S_{BMN}} = \frac{\cancel{\frac{1}{2}} \times AB \times 5\cancel{x} \times \cancel{\sin\hat{B}}}{\cancel{\frac{1}{2}} \times BM \times 3\cancel{x} \times \cancel{\sin\hat{B}}} = 3 \rightarrow \frac{AB}{BM} = \frac{5}{9}$

$\frac{BM}{BM+AM} = \frac{5}{9} \rightarrow \begin{matrix} BM = 5x \\ AM = 4x \end{matrix} \rightarrow \frac{BM}{AM} = \boxed{\frac{5}{4}}$