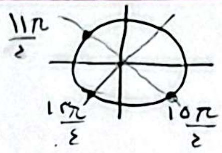
تکلیف سری ۱۸ — (درجه ۲) (صدای) — لعیا کرباسیان

۱۹٫۲۵ عالی

۱- (۱٫۵)

الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4}\cos\frac{13\pi}{4}$

$-1 + \left(-\frac{\sqrt{2}}{2}\right) \times \left(-\frac{\sqrt{2}}{2}\right)$

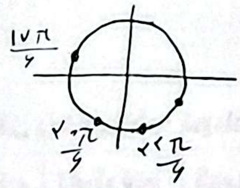


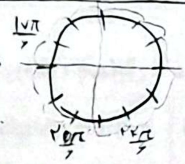
$= -1 + \frac{2}{4} = \frac{-2}{4} = -\frac{1}{2}$ ✓

ب) $\tan\frac{17\pi}{6}\sin\frac{11\pi}{3} + \cos\frac{10\pi}{3}$

$\frac{11\pi}{3} = \frac{22\pi}{6}$

$\frac{10\pi}{3} = \frac{20\pi}{6}$





$\tan\frac{17\pi}{6} \times \sin\frac{11\pi}{3} + \cos\frac{10\pi}{3} = \tan\frac{5\pi}{6} \times \sin\frac{5\pi}{3} + \cos\frac{4\pi}{3}$

$= -\frac{1}{\sqrt{3}} \times (-\sqrt{3}) + \left(-\frac{1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$

۲- (۲)

$\cot x = 4$

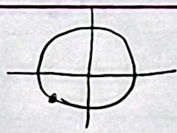
$\frac{\cos x}{\sin x} = 4$

$\cos x = 4\sin x$

$\frac{3\cos x - \sin x}{\cos x + \sin x} = \frac{12\sin x - \sin x}{4\sin x + \sin x} = \frac{11\sin x}{5\sin x} = \frac{11}{5}$ ✓

۳- (۲)

$\sin\alpha = 2\cos\alpha$



$\sin^2\alpha + \cos^2\alpha = 1$

$(2\cos\alpha)^2 + \cos^2\alpha = 1$

$4\cos^2\alpha + \cos^2\alpha = 1 \rightarrow 5\cos^2\alpha = 1$

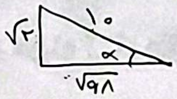
✓ $\cos\alpha = \frac{\sqrt{5}}{5} \xrightarrow{\text{ربع سوم}} \frac{-\sqrt{5}}{5}$

۴- الف) (۱٫۷۵)

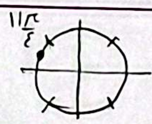
«قسمت ب در آخر فایل قرار دارد»

$\sin\alpha = \frac{\sqrt{2}}{10}$

$\cos\alpha = \frac{\sqrt{98}}{10}$



$\cos\left(\alpha + \frac{11\pi}{4}\right)$



$= \cos\left(\alpha + \frac{3\pi}{4}\right)$

$= \cos\alpha \times \cos\frac{3\pi}{4} + \sin\alpha \times \sin\frac{3\pi}{4}$

$= \frac{\sqrt{98}}{10} \times \frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{10} \times \frac{\sqrt{2}}{2}$ ✓

$= \frac{-14}{20} + \frac{2}{20} = \frac{-12}{20} = \frac{-3}{5}$

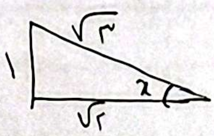
۵- (۲)

$2\sin^2 x + \cos^2 x = \frac{4}{3}$

$-\quad \sin^2 x + \cos^2 x = 1$

$\sin^2 x = \frac{1}{3}$

$\sin x = \frac{1}{\sqrt{3}}$



$\tan^2 x = ?$ $\quad \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$ ✓

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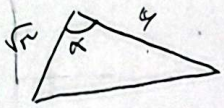
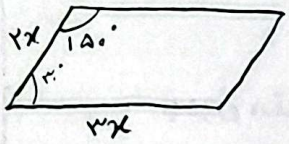


$\frac{1}{2}\sin\alpha \times \sqrt{2} \times 4 = 2\sqrt{2}$

$\frac{\sin\alpha}{2} = \frac{a}{4\sqrt{2}}$

$\sin\alpha = \frac{2}{2\sqrt{2}} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$

$\frac{\max\alpha}{\min\alpha} = \frac{120^\circ}{60^\circ} = 2$ -۶



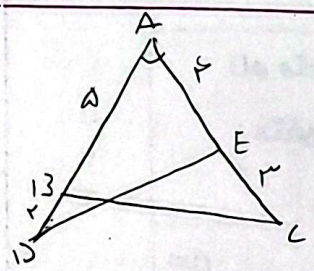
$S = \sin\alpha \times 2x \times 2x$

$S = \frac{1}{2} \times 6x^2$

$24 = 2x^2$

$18 = x^2 \rightarrow x = 3\sqrt{2}$

$P = 4x + 6x = 10x = 30\sqrt{2}$ -۷



$|S_{ABC} - S_{ADE}| = 1,75$

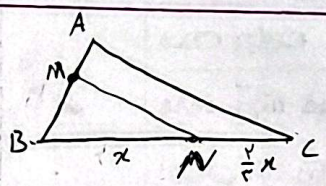
$\left|\sin\hat{A} \times \frac{1}{2} \times 5 \times 7 - \frac{1}{2}\sin\hat{A} \times 4 \times 7\right| = 1,75$

$\frac{35}{2}\sin\hat{A} - \frac{28}{2}\sin\hat{A} = 1,75$

$\sin\hat{A}\left(\frac{7}{2}\right) = 1,75$

$\sin\hat{A} = \frac{1}{2}$ $\cos\hat{A} = \frac{\sqrt{3}}{2}$

$\tan\hat{A} = \frac{1}{\sqrt{3}}$ -۸



$2BN = NC$

$\frac{1}{2}BN = NC$

$\frac{S_{ABC}}{S_{BMN}} = 3 = \frac{\frac{1}{2}\sin\hat{B} \times AB \times 2(x + \frac{2x}{2})}{\frac{1}{2}\sin\hat{B} \times BM \times x}$

$3 = \frac{AB \times \frac{5x}{2}}{BM \times x}$

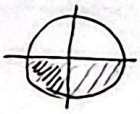
$9\,BM = 5AM + 5BM$

$4BM = 5AM$

$\frac{BM}{AM} = \frac{5}{4}$

$\leftarrow \frac{9}{5} = \frac{(AM + BM)}{BM}$ -۹

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} \rightarrow \frac{|\sin\alpha|}{\cos\alpha} = -\frac{\sin\alpha}{\cos\alpha} \rightarrow |\sin\alpha| = -\sin\alpha \rightarrow \sin\alpha < 0$ -۱۰



$-1 < \sin\alpha < 0$

$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1 + \sin\alpha}{\cos\alpha}$

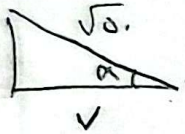
$-2 < 2\sin\alpha < 0$

$-1 < 1 + 2\sin\alpha < 0$

$\frac{1}{|\cos\alpha|} = \frac{1 + 2\sin\alpha}{\cos\alpha} \rightarrow |\cos\alpha| = \frac{\cos\alpha}{1 + 2\sin\alpha}$ ($\cos\alpha > 0$, $1 + 2\sin\alpha < 0$) $\rightarrow \cos\alpha < 0$

$\frac{\cos\alpha}{1 + 2\sin\alpha} > 0$

$\tan\alpha = \frac{1}{2}$ $\quad \sin\left(\frac{13\pi}{4} + \alpha\right)$

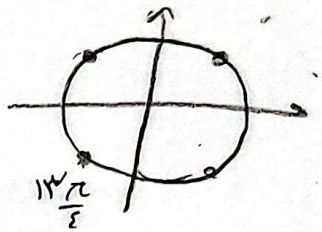


$= \sin\frac{13\pi}{4} \times \cos\alpha + \sin\alpha \times \cos\frac{13\pi}{4}$

$= \frac{-\sqrt{2}}{2} \times \frac{2}{\sqrt{5}} + \frac{-1}{\sqrt{5}} \times \frac{-\sqrt{2}}{2}$

$= \frac{-2\sqrt{2}}{2\sqrt{5}} + \frac{\sqrt{2}}{2\sqrt{5}} = \frac{-\sqrt{2}}{2\sqrt{5}} = -\frac{\sqrt{2}}{\sqrt{10}} = -\frac{1}{5} \times \sqrt{10} = -\frac{\sqrt{10}}{5}$

۴ - ب)



ب) ۱

$$\tan\frac{17\pi}{4} = \tan(4\pi - \frac{\pi}{4}) = -\tan\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\sin\frac{11\pi}{3} = \sin(4\pi - \frac{\pi}{3}) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos\frac{10\pi}{3} = \cos(3\pi + \frac{\pi}{3}) = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$(-\frac{\sqrt{3}}{2})(-\frac{\sqrt{2}}{2}) - \frac{1}{2} = \boxed{0}$$

الف) ۴

$$\cos^2\alpha = 1 - \sin^2\alpha \longrightarrow \cos\alpha = 1 - (\frac{\sqrt{2}}{10})^2 = 1 - \frac{2}{100} = \frac{98}{100} \xrightarrow[\cos\alpha < 0]{\alpha \text{ ناصیه } 2} \cos\alpha = -\frac{7\sqrt{2}}{10}$$

$$\cos(\frac{11\pi}{4} + \alpha) = \cos(3\pi - \frac{\pi}{4} + \alpha) = -\cos(\alpha - \frac{\pi}{4})$$

$$-(\frac{\sqrt{2}}{2}\times\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha) = -\frac{\sqrt{2}}{2}(-\frac{7\sqrt{2}}{10} + \frac{\sqrt{2}}{10}) = -\frac{1}{2}\times -\frac{6\sqrt{2}}{10} = \boxed{\frac{3}{5}}$$