نیکاتابش — دهم دختر B

۱- $\{1\}, \{2,3,4\}, \dots, \underbrace{\{8^2+1, \dots, 9^2\}}_{\text{دسته ۱۹ام}} \Rightarrow b=\frac{a+c}{2}=\frac{8^2+1+9^2}{2}=\frac{162}{2}=81$ (۳)

۲- $\{1,2,3\}, \{4,5,6,\dots 12\}, \{13,14,\dots,39\}, \{40,41,\dots,120\}, \{121,122,\dots,363\} \Rightarrow$ تعداد $= 363-121+1=243$

میانگین $= \frac{S}{\text{تعداد}} = \frac{\frac{363+121}{2}\times 243}{243} = 242$

۳- 2^K; $n=3K \Rightarrow n=0,3,6,9 \Rightarrow 2^0, 2^1, 2^2, 2^3 \Rightarrow 1,2,4,8$

$\begin{cases} -2K+4 ; n=3K+1 \Rightarrow n=1,4,7 \Rightarrow -2(0)+4, -2(1)+4, -2(2)+4 \Rightarrow 4,2,0 \\ \left[\frac{n}{K+2}\right]+a ; n=3K+2 \Rightarrow n=2,5,8 \Rightarrow \left[\frac{2}{0+2}\right]+a, \left[\frac{5}{1+2}\right]+a, \left[\frac{8}{2+2}\right]+a \Rightarrow 1+a, 1+a, 2+a \end{cases}$

$S_{10}=1+2+4+8+4+2+0+1+a+1+a+2+a=19 \Rightarrow 2a+3a=19 \Rightarrow a=-2$

$a_2+a_5+\dots+a_{29} \xrightarrow{n=3K+2} \left[\frac{2}{0+2}\right]+a, \left[\frac{5}{1+2}\right]+a, \dots, \underbrace{\left[\frac{29}{9+2}\right]+a}_{\text{۱۰ جمله } a+2 \text{ هستند}} \Rightarrow 1-2+1-2+8(-2)=$ (-۲)

۴- $an^2+bn+C \Rightarrow a=\frac{1}{10}\times(-14)=-\frac{1}{5}$, $a_5=14, a_7=11/2 \Rightarrow$

$\begin{cases} -\frac{1}{5}(5^2)+5b+c=14 \Rightarrow -5+5(4)+c=14 \Rightarrow c=-1 \\ -\frac{1}{5}(7^2)+7b+c=11/2 \end{cases} \Rightarrow$

$\frac{24}{5}-2b=-3/2 \Rightarrow b=4$

$a_n=-\frac{1}{5}n^2+4n-1 \Rightarrow \frac{a_{15}}{a_1}=\frac{-\frac{1}{5}(15)^2+4(15)-1}{-\frac{1}{5}(1)^2+4(1)-1}=\frac{14}{\frac{14}{5}}=\frac{5\times 14}{14}=$ (۵)

۵- $a_4=a+3d=a'_2, a_8=a+7d=a'_7, a_{10}=0=a'+9d' \Rightarrow a'=-9d'$

$a_8-a_4=4d=a'_7-a'_2=5d' \Rightarrow d=\frac{5}{4}d'$

$\Rightarrow \frac{a'_{15}}{d}=\frac{a'+14d'}{d}=\frac{-9d'+14d'}{\frac{5}{4}d'}=\frac{5d'}{\frac{5}{4}d'}=$ (۴)

* درصورتی که $d\neq 0$ باشد

۶- $2(a+d)^2=5(a+2d)a+3(a+d)a \Rightarrow 2a^2+2d^2+4ad=5a^2+10ad+3a^2+3ad \Rightarrow 0=2a^2-2d^2+ad \Rightarrow$

$(2a-3d)(a+2d)=0 \Rightarrow \begin{cases} a=\frac{3}{2}d \\ a=-2d \end{cases} \Rightarrow \frac{a_4}{d}=\frac{a+3d}{d} \Rightarrow \begin{cases} \frac{\frac{3}{2}d+3d}{d}=\frac{9}{2} \\ \frac{-2d+3d}{d}=1 \end{cases}$

$a, b, c \xrightarrow{\text{هندسی}} b, \frac{a}{2}, \frac{c}{4} \Rightarrow a+d, \frac{a}{2}, \frac{a+4d}{4} \Rightarrow \frac{\frac{a}{2}}{\frac{a+d}{1}} = \frac{\frac{a+4d}{4}}{\frac{a}{2}} \Rightarrow \frac{a}{2a+2d} = \frac{2a+4d}{4a} \Rightarrow 4a^2 = 4a^2 + 16d^2 + 12ad \Rightarrow$ -۷

$a, a+d, a+4d$

$4(2d^2) = 4(-2ad) \Rightarrow d(2d+8a) = 0 \Rightarrow \begin{cases} ① d=0 \text{ ق} \\ ② 2d+8a=0 \Rightarrow a=\frac{-2d}{8} \end{cases}$ چون متمایزند

$\Rightarrow q = \frac{\frac{a}{2}}{a+d} = \frac{\frac{-d}{8}}{\frac{-2d}{8}+d} = \frac{\frac{-d}{8}}{\frac{d}{4}} = -1 \Rightarrow 2q = 2\times(-1) = \boxed{-2}$

$a, b, c \xrightarrow{\text{حسابی}} 8b, 2a, c \Rightarrow 8aq, 2a, aq^2 \Rightarrow aq^2 - 2a = 2a - 8aq \Rightarrow aq^2 + 8aq - 4a = 0 \Rightarrow a(q-1)(q+4) = 0 \Rightarrow \begin{cases} ① a=0 \text{ غ ق ق} \\ ② q=1 \text{ غ ق ق} \\ ③ q=-4 \end{cases}$ -۸

a, b, c

a, aq, aq^2

$\Rightarrow \frac{a_9}{a_4} = \frac{aq^8}{aq^3} = q^5 = (-4)^5 = \boxed{-64}$

$\frac{a_6}{(a_2)^3} + \frac{a_3}{(a)^2} = 2 \Rightarrow \frac{aq^5}{a^3q^3} + \frac{aq}{a^2} = 2 \Rightarrow \frac{q^2+aq}{a^2} = 2 \Rightarrow 2a^2 - q^2 - aq = 0 \Rightarrow (2a+q)(a-q) = 0 \Rightarrow \begin{cases} ① q=-2a \\ ② a=q \end{cases} \Rightarrow$ -۹

$\frac{a^2}{a_2} = \begin{cases} ① \frac{a^2}{aq} = \frac{a^2}{a^2} = \boxed{1} \\ ② \frac{a^2}{aq} = \frac{a^2}{-2a^2} = \boxed{\frac{-1}{2}} \end{cases}$

$a_3 = \sqrt{a_6} \Rightarrow (aq^2)^2 = aq^3 \Rightarrow a^2q^4 = aq^3 \Rightarrow aq = 1 \Rightarrow a_5 = aq^4 = 27 = \cancel{aq} \times q^3 = 3^3 \Rightarrow q=3 \Rightarrow a(3)^4 = 27 \Rightarrow a = \frac{1}{3} \Rightarrow$ -۱۰

$\frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$