

$$\begin{aligned}
 & a, b, c \xrightarrow{\text{فرضی}} b, \frac{a}{r}, \frac{c}{r} \Rightarrow a+d, \frac{a}{r}, \frac{a+rd}{r} \Rightarrow \frac{\frac{a}{r}}{\frac{a+d}{r}} = \frac{a+rd}{\frac{a}{r}} \Rightarrow \frac{a}{ra+rd} = \frac{ra+rd}{ra} \Rightarrow \cancel{ra} = \cancel{ra} + rd + ra \Rightarrow -V \\
 & \downarrow \downarrow \downarrow \\
 & a, a+d, a+rd \\
 & \cancel{r}(rd) = \cancel{r}(-rad) \Rightarrow d(rd+ra) = 0 \Rightarrow \begin{cases} \textcircled{1} d=0 \propto \text{چون متناهیست} \\ \textcircled{2} rd+ra=0 \Rightarrow a = -\frac{rd}{r} \Rightarrow q = \frac{a}{a+d} = \frac{-d}{\frac{-rd}{r} + d} = \frac{-d}{\frac{-rd+d}{r}} = -1 \Rightarrow rq = r \times (-1) = \textcircled{-r} \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 & a, b, c \xrightarrow{\text{فرضی}} rb, ra, c \Rightarrow raq, ra, aq \Rightarrow aq^2 - ra = ra - raq \Rightarrow aq^2 - ca + raq = 0 \Rightarrow a(q-1)(q+c) = 0 \Rightarrow \begin{cases} \textcircled{1} a=0 \propto \text{غیرممکن} \\ \textcircled{2} q=1 \propto \text{غیرممکن} \\ \textcircled{3} q=-c \end{cases} \\
 & \downarrow \downarrow \downarrow \\
 & a, aq, aq^2 \\
 & \Rightarrow \frac{aq}{a_f} = \frac{aq^1}{aq^0} = q^r = (-c)^r = \textcircled{-c^r} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 & \frac{a_f}{(a_f)^r} + \frac{a_r}{(a)^r} = r \Rightarrow \frac{aq^0}{a^r q^r} + \frac{aq}{a^r} = r \Rightarrow \frac{q^r + aq}{a^r} = r \Rightarrow ra^r - q^r - aq = 0 \Rightarrow (ra+q)(a-q) = 0 \Rightarrow \begin{cases} \textcircled{1} q = -ra \\ \textcircled{2} a = q \end{cases} \\
 & \frac{a^r}{a_f} = \begin{cases} \textcircled{1} \frac{a^r}{aq} = \frac{a^r}{a^r} = \textcircled{1} \checkmark \\ \textcircled{2} \frac{a^r}{aq} = \frac{a^r}{-ra^r} = \textcircled{\frac{-1}{r}} \checkmark \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 & a_r = \sqrt{a_f} \Rightarrow (aq^r)^r = aq^r \Rightarrow a^r q^r = aq^r \Rightarrow aq = 1 \Rightarrow a_d = aq^r = rV = aq \times q^r = r^r \Rightarrow q = r \Rightarrow a(r)^r = rV \Rightarrow a = \frac{1}{r} \Rightarrow -10 \\
 & \frac{1}{r} - \frac{1}{r} = \textcircled{\frac{1}{r}} \checkmark
 \end{aligned}$$