

$\{1\} \quad \{2, 3, 4\} \quad \{5, 6, 7, 8, 9\}$
 $\rightarrow 1, 2, 3, 4, 5, 6, 7, 8, 9$
 $t_9 = 1^2 + 1 + 6 + 1 + 6 = 25$
 $\{25, \dots, 81\}$

$\frac{25 + 81}{2} = \frac{106}{2} = 53$

۱

$\rightarrow \{2, 3, 4\}$
 $\rightarrow \{5, 6, 7, 8, 9\}$
 $\rightarrow \{10, 11, \dots, 12\}$
 $\rightarrow \{13, \dots, 14\}$

$\frac{2 + 14}{2} = \frac{16}{2} = 8$

۲

$a_1 = f$
 $a_2 = 1 + d$
 $a_3 = 2$
 $a_4 = 3$
 $a_5 = 1 + d$
 $a_6 = f$
 $a_7 = 0$
 $a_8 = 2 + d$
 $a_9 = 1$
 $a_{10} = -2$

$\rightarrow 8_{10} = 22 + 30 = 52 \rightarrow 30 = -3$
 $\Rightarrow a = -1$
 $\rightarrow a_1 + a_2 + \dots + a_{10} = ?$
 $0 + 2 + 4 + 6 + \dots + 10 = 1,5$
 $\frac{10(1+10)}{2} = 55$

۳

$t_2 = 5d + 2b + c = f$
 $t_7 = 6a + 2b + c = 17$
 $5fa + 2b = 17$
 $\rightarrow 12a + b = 17 \rightarrow b = 17 - 12a$
 $\rightarrow a = \frac{17}{12} = \frac{1}{2}$

$\frac{t_2}{t_1} = \frac{a(1+11)}{a(1+6)} = \frac{12}{7}$

۴

$t_0 = a + 9d = 0$
 $a = -9d$
 $\frac{t_0}{d} = \frac{-9d + 15d}{d} = \frac{6d}{d} = 6$

۵

$$y(a+rd)^r = da(a+rd) + wa(a+rd)$$

$$y(a+rd)^r = da^r + da^r rd + wa^r + wa^r rd$$

$$wa^r + da^r rd = wa^r + da^r rd$$

$$ra^r - rd^r = a^r d^r$$

$$\frac{t_f}{d} = \frac{a+rd}{d}$$

170

1. a, b, c

$a, a, d, a+rd$

2. $b, \frac{a}{r}, \frac{c}{f}$

$$r = \frac{\frac{a}{r}}{b} = \frac{\frac{c}{f}}{\frac{a}{r}} \rightarrow \frac{a}{rb} = \frac{c}{ra} \rightarrow \frac{a}{ra+rd} = \frac{a+rd}{ra} \rightarrow d/rd = \frac{a+rd}{ra}$$

$$r = \frac{\frac{a}{r}}{b} = \frac{a}{ar} = \frac{a}{a+rd} = -1$$

$$r = -r$$

$$\frac{t_f}{t_s} = \frac{ar^r}{ar^r} = r^r = (-r)^r = -r$$

b, ra, c

ra, ra, ar^r

$$ra = ra + ar^r \rightarrow ra = a(r+r^r) \rightarrow r^r + r - r = ar$$

$$r = -r$$

$$\frac{ar^r}{ar^r} + \frac{ar}{ar} = r$$

$$\frac{r}{a} + \frac{r}{a} = r \rightarrow r\left(\frac{r}{a}\right) = r$$

$$\rightarrow \frac{r}{a} = 1$$

$$\frac{ar^r}{ar} = r$$

$$\frac{ar^r}{ar} = \frac{a}{r} = \frac{1}{\frac{r}{a}} = 1$$

$$t_r = \sqrt{t_f}$$

$$ar^r = ar^r$$

$$ar^r = ar^r$$

$$\rightarrow ar = 1$$

$$t_d = ar^r = rv$$

$$ar^r \times r^r = rv \rightarrow r^r = rv$$

$$\rightarrow r = r$$

$$t_r = ar = 1$$

$$t_1 = \frac{1}{r}$$

$$\frac{1}{r} - t_1 = \frac{1}{r} - \frac{1}{r} = \frac{r-r}{r} = 0$$

$$\{1, 2, 3\}, \{4, 5, 6, \dots, 12\}, \{13, 14, \dots, 39\}, \{40, 41, \dots, 120\}$$

۲

$$\boxed{\text{دسته پنجم}} \rightarrow \{121, 122, \dots, 343\}$$

$$\text{میانگین محبات} \rightarrow \bar{x} = \frac{121 + 343}{2} = \frac{464}{2} = \boxed{232}$$

* محبات دسته پنجم دنباله ای صافی با قدر نسبت 1 هستند پس میانگین با میانگین صمد اول را آخر برای مرکز!

۳

$$a_0, a_3, a_4, a_9 \rightarrow \text{ضابطه اول } (2^k)$$

$$a_1, a_6, a_7 \rightarrow \text{ضابطه دوم } (-2k + 6)$$

$$a_2, a_5, a_{11} \rightarrow \text{ضابطه سوم } \left[\frac{n}{k+2} \right] + a$$

$a_2 = \left[\frac{2}{1} \right] + a$ $a_2 = a + 1$
$a_5 = \left[\frac{5}{1} \right] + a$ $a_5 = a + 1$
$a_8 = \left[\frac{1}{2} \right] + a$ $a_8 = a + 2$

$a_0 = 2^0$ $a_0 = 1$	$a_3 = 2^1$ $a_3 = 2$	$a_4 = 2^2$ $a_4 = 4$	$a_9 = 2^3$ $a_9 = 8$
-----------------------	-----------------------	-----------------------	-----------------------

$a_1 = -2(0) + 6$ $a_1 = 6$	$a_6 = -2(1) + 6$ $a_6 = 4$	$a_7 = -2(2) + 6$ $a_7 = 2$
-----------------------------	-----------------------------	-----------------------------

$$S_{10} = 1 + 4 + a_2 + 2 + 2 + a_5 + 4 + 0 + a_8 + 8 = 19$$

$$a_2 + a_5 + a_8 = -2 \rightarrow 3a + 4 = -2 \rightarrow \boxed{a = -2}$$

مجموع محبات $a_2 + a_5 + \dots$ از ضابطه سوم بدست می آید

$$a_n = \left[\frac{3k+2}{k+2} \right] + (-2)$$

$$a_n = \left[\frac{2k+4 + k-2}{k+2} \right] - 2 = \left[2 + \frac{k-2}{k+2} \right] - 2 = \boxed{\left[\frac{k+2}{k-2} \right]}$$

عبارت $\left[\frac{k+2}{k-2} \right]$ به ازای $k=0$ و $k=1$ برابر 1- است دنباله ازای سیر صعودی k برابر

$$S = a_2 + a_5 + a_8 + \dots + a_{29}$$

0 است پس داریم:

$$S = (-1) + (-1) + 0 + \dots + 0 = \boxed{-2}$$

$$t_n = an^r + bn + c \rightarrow t_0 = 2a + b + c = 14 \quad \text{و } t_1 = 4a + b + c = 14, 2$$

$$a = \frac{-1}{v} (-t_0) = \frac{1}{v} (-14) = -\frac{1}{2} \rightarrow 14(-\frac{1}{2}) + b = 14 \rightarrow b = 4$$

$$a = -\frac{1}{2}, b = 4$$

$$t_0 = -14 \rightarrow 2a(-\frac{1}{2}) + 4(1) + c = 14 \rightarrow c = -1$$

$$t_0 = 14^r(-\frac{1}{2}) + 14(1) - 1 = 14$$

$$t_1 = -\frac{1}{2}(1)^r + 4(1) - 1 = 2, 1$$

$$\frac{t_0}{t_1} = \frac{14}{2} = 7$$

دنباله صبی $a_n = a_1 + (n-1)d$ و دنباله صبی $bn \leq cn + k$

$$\begin{aligned} a_k &= br \\ a_n &= bv \\ a_1 + kd &= br \\ a_1 + vd &= bv \end{aligned} \rightarrow kd = \frac{bv - br}{v - k} \rightarrow kd = dc$$

$$b_1 = 0 \rightarrow 1 \cdot c + k = 0 \rightarrow k = -1 \cdot c$$

$$\frac{b_1 a}{d} = \frac{1 \cdot c + k}{d} = \frac{1 \cdot c - 1 \cdot c}{d} = \frac{dc}{d} = \frac{kd}{d} = k$$

$$4(a+d)^r = 2a(a+2d) + 4a(a+d)$$

$$4a^r + 4d^r + 12ad = 1a^r + 12ad$$

$$4a^r + 4d - 4d^r = 0 \rightarrow a = \frac{\pm d + \sqrt{d^r - 4(r)(-4d^r)}}{4} = \frac{-d \pm vd}{4} \rightarrow a = -rd$$

$$1 \rightarrow \frac{a}{d} = \frac{-rd + rd}{d} = 1 \checkmark$$

$$2 \rightarrow \frac{a}{d} = \frac{\frac{rd}{r} + rd}{d} = \frac{4d}{d} = 4, 2 \checkmark$$