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$$\{y_8, \dots, 1\} \rightarrow \frac{y_8 + 1}{2} = \frac{14}{2} = 7$$

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$$\{x, \dots, 2x\} \quad \frac{3x + x}{2} = 2x$$

$$\{1, 2, 3\}, \{2, 3, 4\}, \{3, 4, 5\}, \{4, 5, 6\}, \{5, 6, 7\}, \{6, 7, 8\}, \{7, 8, 9\} \rightarrow \frac{1+2+3}{3} = 2, \frac{2+3+4}{3} = 3, \dots, \frac{7+8+9}{3} = 8$$

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0	1	1+a	2	2	1+a	3	0	2+a	4
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
3k	3k+1	3k+2	3k	3k+1	3k+2	3k	3k+1	3k+2	3k
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
k=0	k=0	k=0	k=1	k=1	k=1	k=2	k=2	k=2	k=3

$$2a + 3a = 19$$

$$a = -2$$

$$a_1 + a_0 + a_1 + \dots + a_9 \rightarrow \text{مجموعه } a_{k+1} \text{ بر مبنای } a_k \\ = a + 1 + a + 1 + a + 2 + a + 2 + \dots + a + 9 \\ = 10a + 18 \quad a = -2 \rightarrow -20 + 18 = -2$$

$$a_0 = 19$$

$$a_n = an^2 + bn + c$$

$$a = -19 \times \frac{1}{10} = -\frac{19}{10}$$

$$a_1 = 14, 2$$

$$a_n = -\frac{1}{10}n^2 + bn + c \rightarrow a_n = -\frac{1}{10}n^2 + 19n - 1$$

$$19 = -\frac{1}{10} \times 1^2 + 19b + c \rightarrow 19 = 19b + c$$

$$14, 2 = -\frac{1}{10} \times 4^2 + 16b + c \rightarrow -16 = 16b + c$$

$$\begin{cases} 19b = 19 \\ b = 1 \\ 19 = 19 + c \\ -1 = c \end{cases}$$

$$\frac{a_{10}}{a_1} = \frac{-\frac{1}{10} \times 10^2 + 19 \times 10 - 1}{-\frac{1}{10} \times 1^2 + 19 \times 1 - 1} = \frac{-10 + 190 - 1}{-0.1 + 18} = \frac{179}{17.9} = 10$$

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$$a_f = t_f$$

$$t_1 = 0 \quad t_1 + 9b = 0$$

$$\frac{t_1 + 14b}{d} = \frac{ab}{d} = \frac{14b}{d} = 14$$

$$a_n = t_n$$

$$a_1 + 3d = t_1 + b \rightarrow a_1 + 3d = -1b$$

$$a_1 + 4d = t_1 + 4b \rightarrow a_1 + 4d = -4b$$

$$a_1 + 4d = -4b$$

$$4a_f = 4a_f a + 3a_f a$$

$$\frac{a_f}{d} = ?$$

$$\frac{a_f + 3d}{d} \xrightarrow{d=1} \frac{1 + (-1)}{-1} = 1$$

$$\begin{cases} a_f = 0 \Rightarrow 4a_f a = 0 \rightarrow a = 0 \\ a_f = 1 \Rightarrow 4a_f a + 3a_f a = 4 \end{cases}$$

$$\frac{a_f}{d} = \frac{0}{1} = 0$$

$$\frac{1 + \frac{1}{0}}{\frac{1}{0}} = 1$$

$$4 \left(\frac{a_f + d}{1-d} \right) \left(\frac{a_f - d}{1-d} \right) + 3 \left(\frac{a_f - d}{1-d} \right) = 4$$

$$4(1-d^2) + 3(1-d) = 4$$

$$4 - 4d^2 + 3 - 3d = 4$$

$$0 = 4d^2 + 3d - 2 \rightarrow d = \frac{-3 \pm \sqrt{9 + 32}}{8} \rightarrow d = \frac{-3 \pm 7}{8} \rightarrow d = 1 \text{ or } d = -\frac{1}{2}$$

$$a, b, c \xrightarrow{+d} a, a+d, a+rd$$

$$b, \frac{a}{r}, \frac{c}{r} \xrightarrow{r} a+d, \frac{a}{r}, \frac{a+rd}{r} \Rightarrow \frac{a^r}{r} = (a+d) \left(\frac{a+rd}{r} \right)$$

$$r = \frac{\frac{a}{r}}{a+d} = \frac{\frac{a}{r}}{a+rd} = \frac{a}{ra+ra} = \frac{a}{-a} = -1$$

$$\frac{ar}{r} = \frac{a^r + rad}{r} + \frac{ad+rd^r}{r}$$

$$ar = ar + rad + rd^r$$

$$-rd = rad \rightarrow d \neq 0$$

$$\boxed{-rd = ra}$$

$$a, b, c \rightarrow a, ar, ar^r$$

$$rb, ra, c \rightarrow rar, ra, ar^r \rightarrow \{ar = rar \times ar^r$$

$$rar = rar^r r$$

$$\frac{r}{r} = r^r$$

$$\frac{ar}{ar} = \frac{ar^r}{ar} = \frac{r^r}{r} = \frac{r}{r} = \frac{r}{r}$$

$$\frac{ar}{(ar)^r} + \frac{ar}{(a)^r} = r \rightarrow \frac{ar^0}{(ar)^r} + \frac{ar}{a^r} = r$$

$$\frac{ar^a}{ar^r r^r} + \frac{ar}{a^r} = r \rightarrow \frac{r^r}{ar} + \frac{r}{a} = r \rightarrow \left(\frac{r}{a}\right)^r + \frac{r}{a} - r = 0 \rightarrow \frac{r}{a} = \frac{-1 \pm \sqrt{1+4r}}{2}$$

$$\frac{a^r}{a_r} = \frac{ar}{ar} = \frac{a}{r} \rightarrow \frac{1}{r}$$

$$ar, a_r, a_d$$

$$\sqrt{a_r}, a_r, r \rightarrow a_r^r = r \sqrt{a_r}$$

$$\downarrow$$

$$\downarrow$$

$$\downarrow$$

$$a_r^r = r \sqrt{a_r}$$

$$a_r^r = r \sqrt{a_r}$$

$$a_r^r = r \sqrt{a_r} \rightarrow \boxed{a_r = 9}$$

$$a, r^r = r$$

$$ra_1 = r$$

$$a_1 = \frac{1}{r} \rightarrow \frac{1}{r} - \frac{1}{r} = \frac{1}{9}$$

$$a, b, c \rightarrow a, aq, aq^r \quad (a, b, c)$$

$$a, b, c \rightarrow raq, ra, aq^r \quad (ra, b, c)$$

$$raq + aq^r = \epsilon a \rightarrow q + q^r = \epsilon$$

$$q = 1 \quad \times$$

$$q = -\epsilon \quad \checkmark$$

$$q = -\epsilon \rightarrow \frac{aq}{aq} = \frac{aq^r}{aq} = q^r = (-\epsilon)^r = -\epsilon$$