①

$\underbrace{\{1\}}_{1^2}, \underbrace{\{2,3,4\}}_{2^2}, \{5,6,7,8,\underbrace{9}_{3^2}\}, \{10,11,12,13,14,15,\underbrace{16}_{4^2}\}, \{17, \ldots, \underbrace{25}_{5^2}\}, \{26, \ldots, \underbrace{36}_{6^2}\}, \ldots$

دسته نهم $\longrightarrow \{\underbrace{8^2+1}_{65}, \ldots, \underbrace{9^2}_{81}\}$

$b = \frac{a+c}{2} \longrightarrow \frac{81+65}{2} = \frac{146}{2} = \boxed{73}$

②

$\{1,2,3\}, \{4,5,\ldots,13\}, \{14,\ldots,39\}, \{40,\ldots,120\}, \{\underline{121},\ldots,\underline{363}\}$

$\frac{363+121}{2} = \frac{484}{2} = \boxed{242}$

③

$2^K \longrightarrow n=3K \longrightarrow 0,3,6,\ldots \quad a_0=1 \quad a_3=2 \quad a_6=4 \quad a_9=8$

$-2K+4 \longrightarrow n=3K+1 \longrightarrow 1,4,7,\ldots \quad a_1=4 \quad a_4=2 \quad a_7=0$

$\left(\frac{n}{K-2}\right)+a \longrightarrow n=3K+2 \longrightarrow 2,5,8,\ldots \quad a_2=1+a \quad a_5=1+a \quad a_8=2+a$

$a_1+a_2+\ldots+a_{10}=19 \longrightarrow 25+3a=19 \longrightarrow a=-2$

$a_2+\ldots+a_{29} \longrightarrow \underbrace{10a+(1+1+\underbrace{2+2+\ldots+2}_{8})}_{18+10a} \longrightarrow 18+10(-2) = \boxed{-2}$

④

$a_5=14 \qquad a_5 \longrightarrow \frac{-1}{5}(25)+5b+c \longrightarrow -5+5b+c=14$

$\hookrightarrow 5b+c=19$

$a_7=17.2 \qquad a_7 \longrightarrow \frac{-1}{5}(49)+7b+c = \frac{-49}{5}+7b+c=17.2$

$\hookrightarrow 7b+c=27$

$\frac{1}{70} \times 14 = \frac{1}{5} \longrightarrow \underbrace{\frac{-1}{5}}_{a}$

$-\begin{matrix} 7b+c \\ 5b+c \end{matrix} \longrightarrow 2b=27-19 \Rightarrow 2b=8 \longrightarrow b=4$

$c=-1$

$\frac{-1}{5}a^2+4a-1 \qquad \frac{a_{15}}{a_1} = \frac{14}{2.8} = \boxed{15}$

⑤

حل $L_n = P_n + q$

$10P+q=0 \longrightarrow q=-10P$

$-10\left(\frac{4}{5}d\right) = -8d$

$L_{15} = 15P+q \longrightarrow 15\left(\frac{4}{5}d\right)-8d \longrightarrow 12d-8d = \boxed{4d}$ برابر

حل $a_8 - a_4 = 4d$

$\left.\begin{matrix} l_7 = a_8 \\ l_2 = a_4 \end{matrix}\right\} l_7 - l_2 = 5P$

$5P = 4d$

$\frac{4}{5}d = P$

$9(a+d)^2 = 5a(a+2d) + 3a(a+4d) \to 9(a^2+2ad+d^2) = 9a^2+18ad+9d^2 = (5a^2+10ad)+3a^2+12ad)$ (6)

$= 8a^2+22ad \to 9a^2+18ad+9d^2 = 8a^2+22ad \to 2a^2 + ad - 6d^2 = 0$

$\xrightarrow[d^2]{\text{تقسیم بر}} \frac{2a^2}{d^2} + \frac{a}{d} - 6 = 0 \to \Delta = 1+48 = \sqrt{49} \to \frac{-1+7}{4} = \frac{6}{4}$

$\frac{-1-7}{4} = -2$

$\frac{a_4}{d} = \frac{a+3d}{d} = \frac{-2+3}{1} = 1$ $\frac{a_4}{d} = \frac{a+3d}{d} = \frac{\frac{3}{2}+3}{1} = 4{,}5$

واسطه حسابی: $b = \frac{a+c}{2} \to a = 2b - c$ (7)

واسطه هندسی: $\frac{a^2}{4} = \frac{bc}{4} \to a^2 = bc$ $\to (2b-c)^2 = bc$

$4b^2 - 5bc + c^2 = 0 \to (b-c)(4b-c) = 0 \to \begin{cases} b=c & \text{غ ق ق} \\ 4b=c \end{cases}$ متمایز نیستند

واسطه حسابی: $\frac{a+4b}{2} \to b \to a = -2b$

هندسی: $b, -b, b \to q = -1 \to 2q = -2$

$b = cr^4$ (8)

$a = cr^2$

$2a = \frac{c+3b}{2} \to 4a = c+3b \to 4cr^2 = c + 3cr \xrightarrow{\text{تقسیم بر } c} 4r^2 - 3r - 1 = 0$

$\Delta = (-3)^2 + 16 = 25$

$\frac{3+5}{8} = 1$ غ ق ق $\frac{3-5}{8} = \frac{-1}{4}$

چون جملات متمایز

$\frac{a_5}{a_4} = r^3 \to \left(\frac{-1}{4}\right)^3 = \frac{-1}{64}$

$\frac{aq^5}{(aq)^3} + \frac{aq}{a^2} = 2 \to \frac{q^2}{a^2} + \frac{aq}{a^2} = 2 \to 2a^2 = q^2 + aq \to 2a^2 - aq - q^2 = 0 \to \underbrace{(q-a)}_{q=a}\underbrace{(q+2a)}_{q=-2a} = 0$ (9)

$\frac{a_2}{a_3} = \frac{a_2}{-2a_2} \to 1, \frac{-1}{2}$

$ar^2 = \sqrt{ar^3}$ (10)

$a^2r^4 = ar^3 \to ar = 1 \to a = \frac{1}{r}$

$a_5 = ar^4 = 27 \to \frac{1}{r} \times r^4 = r^3 = 27 \to r = 3$

$a = \frac{1}{r} = \frac{1}{3}$

$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$