دهم دختر B

نیکا تابش

۱-

$\frac{a}{q}, a, aq, \dots \Rightarrow \frac{a}{q}\times a\times aq = 64 \Rightarrow a^3=64 \Rightarrow a=4 \Rightarrow \frac{a}{q}+a+aq=21 \Rightarrow \frac{4}{q}+4+4q=21 \Rightarrow \frac{4}{q}+4q-17=0 \Rightarrow 4q^2-17q+4=0$

$\Rightarrow q=\frac{-(-17)\pm\sqrt{(-17)^2-4(4\times 4)}}{2(4)} \Rightarrow q=\frac{17\pm 15}{8} \Rightarrow \begin{cases} ①\ q=4 \\ ②\ \boxed{q=\frac{1}{4}} \end{cases}$

چون حسابی نیست

پس این قابل قبول است

۲-

$(2x)^2=(x^2-2)(x^2+4) \Rightarrow 4x^2=x^4+2x^2-8 \Rightarrow 2x^2=x^4-8 \Rightarrow x^4-2x^2-8=0 \Rightarrow (\underbrace{x^2-4}_{x^2=4})(x^2+2)=0$

$x=\pm 2 \Rightarrow \left.\begin{array}{l} x=2 \Rightarrow 8, 4, 2 \Rightarrow q=\frac{1}{2} \\ x=-2 \Rightarrow 8, -4, 2 \Rightarrow \text{غ ق ق} \quad (x=\frac{-1}{2}) \end{array}\right\} \Rightarrow$

$S_7 = 8\times\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}} = 8\times\frac{1-\frac{1}{128}}{\frac{1}{2}} = \boxed{\frac{127}{8}}$

۳-

$1+q+q^2+q^3+q^4=\frac{121}{81},\ S_5=a_1+a_1q+a_1q^2+a_1q^3+a_1q^4=a_1(1+q+q^2+q^3+q^4)=243\times\frac{121}{81}=\boxed{363}$

۴-

حسابی: $A=\frac{1+64}{2}=32.5$ ، هندسی: $B^2=1\times 64 \Rightarrow B=\pm 8 \Rightarrow \begin{cases} ①\ A+B=32.5+8=\boxed{40.5} \\ ②\ A+B=32.5-8=\boxed{24.5} \end{cases}$

۵-

$a_{101}=a_1+100d,\ d=\frac{-95}{4}-(-24)=\frac{1}{4} \Rightarrow a_{101}=-24+100\times\frac{1}{4}=1$

$128, a_2, \dots \Rightarrow a_n=128\times q^{n-1} \Rightarrow a_8=128\times q^7=1 \Rightarrow \boxed{q=\frac{1}{2}}$

۶-

a_3, a_7, a_9

$a_1+2d, a_1+6d, a_1+8d \Rightarrow (a_1+6d)^2=(a_1+2d)(a_1+8d) \Rightarrow a_1^2+36d^2+12a_1d=a_1^2+16d^2+10a_1d \Rightarrow 20d^2=-2a_1d \Rightarrow 2d(10d+a_1)=0$

$\Rightarrow ①\ 2d=0 \Rightarrow d=0$ غ ق ق

$②\ 10d+a_1=0 \Rightarrow \boxed{d=\frac{-a_1}{10}}$

۷-

a_2, a_4, a_8

$a_1+d, a_1+3d, a_1+7d \Rightarrow (a_1+3d)^2=(a_1+d)(a_1+7d) \Rightarrow a_1^2+9d^2+6a_1d=a_1^2+7d^2+8a_1d \Rightarrow 2d^2=2a_1d \Rightarrow d=a_1 \Rightarrow 2a, 4a, 8a \Rightarrow$

$q=\frac{4a}{2a}=2 \Rightarrow a_{10}=a_1q^9=\frac{1}{4}\times 2^9=\boxed{128}$

۸-

$3a_2, 2a_3, a_4$

$a_1q=0 \Rightarrow a_1=0$ غ ق ق

$3(a_1q), 2(a_1q^2), a_1q^3 \Rightarrow 2a_1q^2=\frac{3a_1q+a_1q^3}{2} \Rightarrow -4a_1q^2+3a_1q+a_1q^3=0 \Rightarrow a_1q(-4q+3+q^2)=0 \Rightarrow (q-1)(q-3)=0 \Rightarrow ①\ q=1$ غ ق ق

$②\ \boxed{q=3}$

۹-

$d=\frac{7}{4}-2=\frac{-1}{4} \Rightarrow \left.\begin{array}{l} a_4=2+3\times(\frac{-1}{4})=\frac{5}{4} \\ a_8=2+7\times(\frac{-1}{4})=\frac{1}{4} \\ a_{13}=2+12\times(\frac{-1}{4})=-1 \end{array}\right\} \Rightarrow \frac{5}{4}+x, \frac{1}{4}+x, -1+x \Rightarrow (\frac{1}{4}+x)^2=(x+\frac{5}{4})(x-1) \Rightarrow$

$\frac{1}{16}+x^2+\frac{1}{2}x=x^2+\frac{1}{4}x-\frac{5}{4} \Rightarrow x=\frac{-21}{4} \Rightarrow -4, -5, \frac{-25}{4} \Rightarrow q=\frac{\frac{-25}{4}}{\frac{-5}{1}}=\boxed{\frac{5}{4}}$

۱۰ـ

$$a_1 + a_1 q^3 + a_1 q^6 = 73$$

$$t_1 = a_1 = a ,\ t_2 = a_4 = aq^3 ,\ t_{10} = a_7 = aq^6$$

$$\left.\begin{array}{l} t_2 = aq^3 \\ t_2 = t_1 + d \end{array}\right\} \Rightarrow aq^3 = a + d \Rightarrow d = a(q^3 - 1)$$

$$t_{10} = aq^6 = a + 9a(q^3-1) = a + 9aq^3 - 9a \Rightarrow aq^6 - 9aq^3 + 8a = 0 \Rightarrow a(q^6 - 9q^3 + 8) \Rightarrow q^3 = \frac{9 \pm \sqrt{81-32}}{2} \Rightarrow \left\{\begin{array}{l} 1)\ q^3 = 8 \Rightarrow q = 2 \\ 2)\ q^3 = 1 \Rightarrow q = 1 \end{array}\right\} \Rightarrow$$

$$q = 1 \Rightarrow a + a + a = 73 \Rightarrow a = \frac{73}{3} \Rightarrow d = a(q^3-1) = \frac{73}{3} \times (1-1) = \boxed{0}$$

$$q = 2 \Rightarrow a + 8a + 64a = 73 \Rightarrow a = 1 \Rightarrow d = a(q^3-1) = 1 \times (8-1) = \boxed{7}$$