نام و نام خانوادگی: تانیا ترک ترابی　　　سری: ۱۵　　　گروه: دهم یکشنبه

۱۹٫۵ آفرین

۱

$\frac{a}{q} + a + aq = 21 \Rightarrow \frac{4}{q} + 4 + 4q = 21 \Rightarrow \frac{4}{q} + 4q = 17 \Rightarrow 4q^2 - 17q + 4 = 0 \Rightarrow$

$q^2 - 17q + 16 = 0 \Rightarrow (q-1)(q-16)$

$\frac{a}{q} \times a \times aq = 64 \Rightarrow a^3 = 64 \Rightarrow a = 4$

$\boxed{q = \frac{1}{4}}$, $q = 4$ ✗ ✓

(۲)

۲

$x^2+4 , 2x , x^2-2$　　$(2x)^2 = (x^2+4)(x^2-2) \Rightarrow 4x^2 = x^4 + \underbrace{2x^2 + 4x^2}_{2x^2} - 8 \Rightarrow$

① $x = 2$　$8 , 4 , 2$　$q = \frac{1}{2}$ ✓　$4x^2 = x^4 + 2x^2 - 8 \Rightarrow x^4 - 2x^2 - 8 = 0$　$y = x^2$

② $x = -2$　$8 , -4 , 2$　$q = -\frac{1}{2}$ ✗　$y = \frac{2 \pm \sqrt{4+32}}{2} = \frac{2 \pm 6}{2} \Rightarrow y_1 = 4 , y_2 = -2$ ✗　$x = \pm 2$

$S_7 = a\,\frac{1-q^n}{1-q} \Rightarrow 8 \times \frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}} \Rightarrow S_7 = \frac{127}{8}$ ✓

(۲)

۳

$1 + q + q^2 + q^3 + q^4 = \frac{121}{81} \Rightarrow 1 \times \frac{1-q^5}{1-q} = \frac{121}{81} \Rightarrow q = \frac{1}{3}$

$S_n = a_1 \frac{1-q^n}{1-q} \Rightarrow S_5 = 243 \times \frac{1-(\frac{1}{3})^5}{1-\frac{1}{3}} \Rightarrow S_5 = \frac{\frac{242}{243}}{\frac{2}{3}} \Rightarrow S_5 = 363$ ✓

(۲)

۴

$1 , \underline{2} , \underline{4} , \underline{8} , \underline{16} , \underline{32} , 64$　　$q^{n+1} = \frac{b}{a} \Rightarrow q^6 = 64 \Rightarrow q = \pm 2$

(8 ↓ B)

$1 , \underline{11.5} , \underline{22} , \underline{32.5} , \underline{43} , \underline{53.5} , 64$　　$d = \frac{b-a}{n+1} \Rightarrow \frac{63}{6} = 10.5$ ✓

(32.5 ↓ A)

$A + B = 32.5 + 8 = 40.5$ ✓

(۲)

۵

$-24 , -\frac{95}{4} , \ldots$　　$d = -\frac{95}{4} - (-24) = -\frac{95}{4} + \frac{96}{4} = \frac{1}{4}$

$a_{101} = -24 + 25 = 1$

$128 , a_2 , \ldots$　　$a_8 = 1 \Rightarrow 1 = 128 \times q^7 \Rightarrow q^7 = \frac{1}{128} \Rightarrow \boxed{q = \frac{1}{2}}$ ✓

(۲)

۶

$a_1+2d,\ a_1+6d,\ a_1+8d$ $\quad (a_1+6d)^2=(a_1+2d)(a_1+8d) \Rightarrow$

$a^2+12ad+36d^2=a^2+\underbrace{8ad+2ad}_{10ad}+16d^2 \Rightarrow$

$2ad+20d^2=0 \Rightarrow 2d(a+10d)=0 \Rightarrow d=0$

$a=-10d \Rightarrow$

$\frac{a}{-10}=d$

۷

$\underbrace{a_1+d}_{\frac{1}{2}},\ \underbrace{a_1+3d}_{\frac{1}{2}},\ \underbrace{a_1+7d}_{1}$ $\quad (a_1+3d)^2=(a_1+7d)(a_1+d) \Rightarrow$

$a^2+6ad+9d^2=a^2+\underbrace{ad+7ad}_{8ad}+7d^2$

$9d^2-7d^2=8ad-6ad \Rightarrow 2d^2=2ad \Rightarrow$

$2d(d)=2d(a) \Rightarrow d=a$

$a_1+d=\frac{1}{2}$

$2a=\frac{1}{2} \Rightarrow a=\frac{1}{4}$

$q=2$

$aq^9=\frac{1}{4}\times(2)^9=\frac{1}{4}\times 512=128$ ✓

۸

$2aq,\ 2aq^2,\ aq^3 \Rightarrow 2aq^2=\frac{2aq+aq^3}{2} \Rightarrow 4aq^2=\underbrace{2aq+aq^3}_{aq(2+q^2)}$

$2q(2q)=q(2+q^2) \Rightarrow 2q=2+q^2$

$q^2-4q+2=0 \Rightarrow (q-1)(q-2) \Rightarrow \boxed{q=2},\ q=1$ ✗

۹

$2,\ \frac{7}{4},\ \ldots \quad d=\frac{7}{4}-\frac{8}{4}=-\frac{1}{4} \quad a_4=\frac{5}{4} \quad a_8=\frac{1}{4} \quad a_{12}=-1$

$\frac{5}{4}+k,\ \frac{1}{4}+k,\ -1+k \quad \left(\frac{1}{4}+k\right)^2=\left(-1+k\right)\left(\frac{5}{4}+k\right) \Rightarrow -\frac{5}{4}-k+k^2+\frac{5}{4}k=k^2+\frac{1}{4}k-\frac{5}{4}$

$k^2+\frac{1}{2}k+\frac{1}{16}=k^2+\frac{1}{4}k-\frac{5}{4} \Rightarrow \frac{1}{4}k=-\frac{21}{16} \Rightarrow k=-\frac{21}{4}$

$q=\frac{a_{12}+k}{a_8+k} \Rightarrow \frac{-1-\frac{21}{4}}{\frac{1}{4}-\frac{21}{4}}=\frac{-25}{-20}=\frac{5}{4} \Rightarrow \boxed{q=\frac{5}{4}}$ ✓

$G_1=A_1$
$G_4=A_2$
$G_7=A_{10}$

۱۰

$a_1=a \quad aq^{11}=a+4d \quad aq^9=a+9d$

$a_1+aq^2+aq^9=72 \Rightarrow a(1+q^2+q^9)=72 \quad (aq^3)^2=a(aq^9)=a^2q^9$ (۱٫۵)

$d=aq^3-a$

$9d=aq^9-a$

$9(aq^3-a)=aq^9-a \Rightarrow 8a=9aq^3-aq^9 \Rightarrow 8a=9aq^3-aq^9 \Rightarrow$

$8=9q^3-q^9 \Rightarrow x^2-9x+8=0 \Rightarrow (x-1)(x-1)=0 \Rightarrow q^3=8 \to q=2$

$q^3=1 \to q=1$ ✗

$a_1(1+8+64)=72 \Rightarrow a_1=1 \quad a_5=aq^4=1\times 8=8 \Rightarrow \boxed{d=8}$ ✓

۱۰

جملات لزوماً متمایز هستند چون می‌توانند ثابت باشند!

$$q=1 \rightarrow 3a=\nu\mathfrak{r} \rightarrow a=\frac{\nu\mathfrak{r}}{3}$$

$$aq^{2}-a=d \xrightarrow{q=1} \frac{\nu\mathfrak{r}}{3}-\frac{\nu\mathfrak{r}}{3}=0 \rightarrow d=0$$