باران عسگری دهم ریاضی B    سمر ۱۴۰۵    ۱۹٫۷۵ عالیه ۱۱

۱)

$a_1 = \frac{1}{2} , q = 2$    ۲

الف) $a_{10} = \frac{1}{2} \times 2^9 = \boxed{256}$    ج) $b^2 = ac \to \sqrt{4 \times 256} = \sqrt{2^{10}} = 2^5 = \boxed{32}$

ب) $\frac{a_{17}}{a_{13}} = q^{17-13} = q^4 = 2^4 = \boxed{16}$

د) $a_n = a_1 q^{n-1} \to 128 = \frac{1}{2} \times 2^{n-1} \Rightarrow \underbrace{256}_{2^8} = 2^{n-1} \Rightarrow n-1 = 8$, $n = 9$

۲)

$q^{n-m} = \frac{a_n}{a_m} \to q^{8-5} = \frac{96}{12} \Rightarrow q^3 = 8 \Rightarrow \boxed{q = 2}$    ۲

$a_n = a_m \times q^{n-m} \Rightarrow a_{10} = a_8 \times 2^2 \Rightarrow a_{10} = 96 \times 4 = \boxed{384}$

۳)

الف) $\left(\frac{a_3}{q^2}\right) \times \left(\frac{a_3}{q}\right) \times (a_3) \times (a_3 q) \times (a_3 q^2) = a_3^5 = \underbrace{243}_{3^5} \Rightarrow \boxed{a_3 = 3}$    ۲

ب) $\frac{3}{\cancel{q^2}} \times (3\cancel{q^2}) = \boxed{9}$

۴)

$4\sqrt{2} = 2^{\frac{5}{2}} , b^2 = ac \to (2^{\frac{5}{2}})^2 = 2^a \times 2^b \Rightarrow 2^5 = 2^a \times 2^b$    ۲

$\Rightarrow 2^5 = 2^{a+b} \Rightarrow a+b = 5 \Rightarrow \frac{a+b}{2} = \frac{5}{2}$

۵)

$11 - 7 = 4 \qquad 7 - 3 = 4$    ۱٫۷۵

$\to x^2 = (1-x)(x+1) \Rightarrow x^2 = 1 - x^2 \Rightarrow x = \pm\sqrt{\frac{1}{2}}$

$\to \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\pm\sqrt{2}}{2}}$

به ازای $x = -\frac{\sqrt{2}}{2}$ حاصل $\frac{a_{11}}{a_7} = q^4$ عددی منفی میشد در صورتی که $q^4$ قطعا نامنفی است پس $x = -\frac{\sqrt{2}}{2}$ غیر قابل قبول است

۶)

$a_1 + a_1 q^3 = 28 \to a_1(1+q^3) = 28$    $\frac{a_1(1+q^3)}{a_1 q(1+q)} = \frac{28}{12} = \frac{7}{3}$    ۲

$a_1 q + a_1 q^2 = 12 \to a_1 q(1+q) = 12$

$\Rightarrow \frac{(1+q)(1-q+q^2)}{q(1+q)} = \frac{7}{3} \Rightarrow 3 - 3q + 3q^2 = 7q \Rightarrow 3q^2 - 10q + 3 = 0$

$q = 3 \to 3a_1(1+3) = 12 \Rightarrow a_1 = 1$

$\Rightarrow q \overset{\Delta}{=} \frac{-10 \pm \sqrt{64}}{6}$

$q = \frac{1}{3} \to a_1 q(1+q) = 12 \Rightarrow a_1 = 27$

$q = 3 \Rightarrow 1, 3, 9, 27$ / $q = \frac{1}{3} \Rightarrow 27, 9, 3, 1$

(۷)

$$\frac{a_2}{q} \times a_2 \times a_2 q = 27 \Rightarrow a_2^3 = 27 \Rightarrow a_2 = 3 \to a_1 = \frac{3}{q}$$

$$\to a_1 + a_2 + a_3 = 13 \xrightarrow{a_2=3} a_1 + a_3 = 10 \Rightarrow a_1(1+q^2) = 10 \to \frac{3}{q}(1+q^2) = 10$$

$$\Rightarrow \frac{3}{q} + 3q = 10 \xrightarrow{\times q} 3 + 3q^2 - 10q = 0 \Rightarrow 3q^2 - 10q + 3 = 0$$

$$\Rightarrow q = \frac{10 \pm 8}{6} < \begin{matrix} 3 \\ \frac{1}{3} \end{matrix} \quad , q = 3 \to 1, 3, 9$$

$$q = \frac{1}{3} \to 9, 3, 1$$

(۸)

الف) $S_{10} = 2\left(\frac{3^{10}-1}{3-1}\right) = 59048$

ب) $P_n = (a_1 \times a_n)^{n}$ یا $\to a_n$ جایگذاری $\Rightarrow P_n = a_1^n \times q^{\frac{n(n-1)}{2}}$

$$\Rightarrow 2^{10} \times 3^{\frac{10(10-1)}{2}} = 2^{10} \times 3^{45} \Rightarrow 1024 \times 3^{45}$$

(۹)

$$A = a_1, a_1 q, a q^2, \ldots a q^{n-1}$$

$$B = \to b_1 = a_2 - a_1 = aq - a \quad \Rightarrow b_n = a_{n-1} - a_n = aq^n - aq^{n-1}$$

$$b_2 = a_3 - a_2 = aq^2 - aq$$

$$\hookrightarrow \frac{b_2}{b_1} = \frac{b_3}{b_2} \to \frac{aq(q-1)}{a(q-1)} = q$$

$$\boxed{\frac{b_3}{b_2} = q}$$

(۱۰)

الف) $S_n = a_1 + (a_1+d) + (a_1+2d) + \ldots + (a_1+(n-1)d)$ ★★

نزولی $\hookrightarrow S_n = a_n + (a_n - d) + (a_n - 2d) + \ldots + (a_1+d) + a_1$ ★

$$\Rightarrow S_n = (a_1 + (n-1)d) + (a_1 + (n-2)d) + \ldots + a_1$$

$$★ + ★★ \Rightarrow 2S_n = (2a_1 + (n-1)d) + (2a + (n-1)d) + \ldots$$

$$2S_n = n(2a_1 + (n-1)d) \Rightarrow S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

ب) $S_n = a_1 + aq + aq^2 + \cdots + aq^{n-1}$

$\xrightarrow{\times q} aq + aq^2 + \cdots + aq^n$

$\Rightarrow qS_n - S_n = aq^n - a$

$S_n(q-1) = a(q^n - 1) \Rightarrow S_n = a\dfrac{q^n - 1}{q - 1}$