(عارفه سیفی - دهم دختر)

۱) الف) ← $\frac{1}{2}, 1, 2, 4$ ← $t_n = 2^{n-2}$ ← $t_{10} = 2^8$

ب) ← $\frac{a_{17}}{a_{13}} = \frac{2^{15}}{2^{11}} = \underline{2^4}$

ج) ← $b^2 = ac$ ← $2^2 \times 2^8 = 2^{10}$ ← $\underline{b = 2^5}$

د) ← $t_n = 128$ ← $n = ?$ ← $2^{n-2} = 2^7$ ← $n-2=7$ ← $\underline{n=9}$

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۲) $t_5 = 12$ ، $t_8 = 96$ ، $t_{10} = ?$

$t_n = t_1 q^{n-1}$

$t_5 = t_1 q^4 = 12$

$t_8 = t_1 q^7 = 96$ } → $q^3 = 8$ → $\underline{q = 2}$

→ $t_1 (2)^4 = 12$ → $\underline{t_1 = \frac{3}{4}}$

$t_{10} = \frac{3}{4}(2)^9 = \underline{3 \times 2^7}$

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۳) $243 = a_1 \times a_2 \times a_3 \times a_4 \times a_5$ ←

$a_1 \times a_5 = a_3 \times a_3$

$a_2 \times a_4 = a_3 \times a_3$

→ $a_3^5 = 243$ → $a_3 = \sqrt[5]{243}$

الف) ← $\sqrt[5]{243}$

ب) ← $a_1 \times a_5 = a_3^2$ ← $\frac{(\sqrt[5]{243})^2}{}$ → $\underline{243^{2/5}}$

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۴)

$\overbrace{t_1}^{1} + \overbrace{t_1 q}^{3} + t_1 q^2 = 13$

→ ~~$t_1(1+q+q^2) = 13$~~

→ $\frac{3}{q}(1+q+q^2) = 13$

→ $\frac{(1+q+q^2)}{q} = \frac{13}{3}$

→ $\underline{q = 3}$ → $\underline{t_1 = 1}$

$\underbrace{t_1 \times t_1 q \times t_1 q^2}_{} = 27$

$t_1^3 \times q^3 = 3^3$ → $(t_1 \times q)^3 = 3^3$ → $\underline{t_2 = 3}$

$t_1^3 = \frac{27}{q^3}$ → $t_1 = \frac{3}{q}$

→ $1 + 3 + t_3 = 13$ → $\underline{t_3 = 9}$

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۵) $a_n = 2, 6, 18, \ldots$ ، $t = 2$ ، $q = 3$

$S_n = a_1\left(\frac{q^n - 1}{q-1}\right)$

الف) $S_{10} = 2\left(\frac{3^{10}-1}{3-1}\right) = \underline{3^{10}-1}$

$P_n = \sqrt{(a_1 \times a_n)^n}$

$a_{10} = \underline{2(3)^9}$

ب) $P_{10} = \sqrt{(2^2 \times 3^9)^{10}} = (2^2 \times 3^9)^5 = \underline{2^{10} \times 3^{45}}$

۹)

$t_1 ، t_1 q ، t_1 q^2 ، \ldots$

$\hookrightarrow t_1 q - t_1 = t_1(q-1)$

$$\left.\begin{array}{l} t_1 q - t_1 = t_1(q-1) \\ t_1 q^2 - t_1 q = t_1 q(q-1) \\ \quad\vdots \end{array}\right\} \rightarrow t_1(q-1) \xrightarrow{\times q} t q(q-1) \xrightarrow{\times q} t q^2(q-1), \ldots$$

* دنباله هندسی است.
* هر جمله بر حسب q جمله اول به دست می‌آید

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۱۰)

ب)

$$-\begin{cases} S_n = a_1 + a_1 q + a_1 q^2 + \ldots + a_1 q^{n-1} \\ S_n q = a_1 q + a_1 q^2 + a_1 q^3 + \ldots + a_1 q^n \end{cases} \times q$$

تفریق

$S_n - S_n q = a_1 - a_1 q^n$ فاکتورگیری

$S_n(1-q) = a_1(1-q^n)$

$\hookrightarrow S_n = a_1\left(\dfrac{1-q^n}{1-q}\right)$

الف)

$S_n = t_1 + (t_1 + d) + (t_1 + 2d) + \ldots + \overset{t_n}{\cancel{(t_1 + (n-1)d)}}$

$= \overset{t_n}{\cancel{(a_1 + (n-1)d)}} + (t_n - d) + (t_n - 2d) + \ldots + (t_1 + d) + t_1$

جمع

$\hookrightarrow 2S_n = (t_1 + t_n) + (t_1 + t_n) + (t_1 + t_n) + \ldots + (t_1 + t_n)$

$2S_n = n(t_1 + t_n) \rightarrow S_n = \dfrac{n(t_1 + t_n)}{2} \rightarrow \dfrac{n}{2}\left(\overset{2t_1 + (n-1)d}{\cancel{t_1 + t_1 + (n-1)d}}\right)$

$S_n = \dfrac{n}{2}\left(2t_1 + (n-1)d\right)$